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# Walter Christaller From “exquisite corpse” to “corpse resuscitated”



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Surveys

# Walter Christaller From "exquisite corpse" to "corpse resuscitated"

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Abstract

*In most currently available geography books, spatial representations group sets of differentiated location-objects, which can be located (directly or indirectly) on the surface of the Earth, using latitude, longitude and altitude, and systems projecting this surface on a map. But in fact spaces defined with the help of cartographic projection systems are independent of the locations-objects which are represented there. That being so, once the location-object is represented with the aid of a projection space, the cartographic spaces which have been generated can combine the locations-objects so that they can be seen as geometrizations, giving rise to geovisualizations. But these geo-visualo-metrizations—presumed to be objective—can be used to formulate geo-interpretations, determined on the one hand by the a priori choice the observer made of a projection system and, on the other hand, by beliefs and ideologies expressed with the aid of explicit or implicit geovisions.*

*One of the best-known geo-interpretations is the ideal image proposed by Walter Christaller in 1933, in which he claims to explain the central function of a location-object on the surface of the Earth, using a geometrization of its location in a regular triangular-hexagonal system. However, the initial geometric diagram that Walter Christaller used to solve the problem he raised is mathematically unsound.*

*For Walter Christaller's direct followers, this theory is still valid and it is possible to use it to construct "models" which remain "useful" using amputations or grafts, despite the fact that one of the main components has been proven wrong by a description of reality. The "exquisite corpse" method consists in putting together ideas considered to be "true", with ideas that are known to be false, in the belief that the true will cancel out the "false" and make them come "true".*

*This so-called "theory" was salvaged, by neglecting or obliterating three quarters of a century's worth of contradiction between observation and theoretical postulates, by dint of erasing and censoring Die zentralen Orte in Süddeutschland, by moving away from or simplifying the ideal triangular-hexagonal "explanations", by unjustifiably bestowing diagrams by other authors upon Walter Christaller, by inversing the logic of the "central places system" and, finally, proposing contradictory geometric interpretations of its principles. The amputation and graft process has continued without interruption since the end of World War II, more or less intensively at various times depending on the geographic linguistic areas.*

*The view that this geometrization was objective has encouraged and consolidated ideological geo-interpretations based on a central hexagon representation, and a "geovision" has emerged based on authority and utility and the idea of "center" has become a toxic geographic concept.*

**Keywords:** Geography, interpretations, geovisualizations, geometrizations, spatial, representations, concept, Walter Christaller

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In most currently available geography books, spatial representations group sets of differentiated location-objects, which can be located (directly or indirectly) on the surface of the Earth, using latitude, longitude and altitude, and systems projecting this surface on a map. But in fact spaces defined with the help of cartographic projection systems are independent of the locations-objects which are represented there. That being so, once the location-object is represented with the aid of a projection space, the cartographic spaces which have been generated can combine the locations-objects so that they can be seen as geometrizations, giving rise to geovisualizations. But these geo-visualo-metrizations—presumed to be objective—can be used to formulate geo-interpretations, determined on the one hand by the a priori choice the observer made of a projection system and, on the other hand, by beliefs and ideologies expressed with the aid of explicit or implicit geovisions. The geovisions can be used to generate in turn new geometrizations which may, or may not, stem from scientific use of the results of object representation using cartographic projection systems. As a result, although they are made using, initially, maps arising out of conventional cartography or geomatics following data interpretation, the geo-visualo-metrizations of results can be shown on geomaps which are geovisions of geo-interpretations (figure 1).

## 1. A SO CALLED "MODEL BASED ON AN INITIAL MATHEMATICAL ERROR

One of the best-known geo-interpretations is the ideal image generated by the geovision of centrality proposed by Walter Christaller in 1933, in which he claims to explain the central function of a location-object on the surface of the Earth, using a geometrization of its location in a regular triangular-hexagonal system. Figure 2 illustrates the way in which he sets the problem out. However, the initial geometric diagram that Walter Christaller used to solve the problem he raised is mathematically unsound.

The first of his "principles" is the "market principle". This is supposed to be the result of economic laws of supply and demand. The "central place", situated at the vertex of a hexagon and considered to be a town, is a place where goods are created and consumed. The more goods and services a town has to offer, the more its "sphere of influence" as a "central place" is extensive. People tend to congregate and collect there. Apart from itself, each "central place", situated at the centre of a

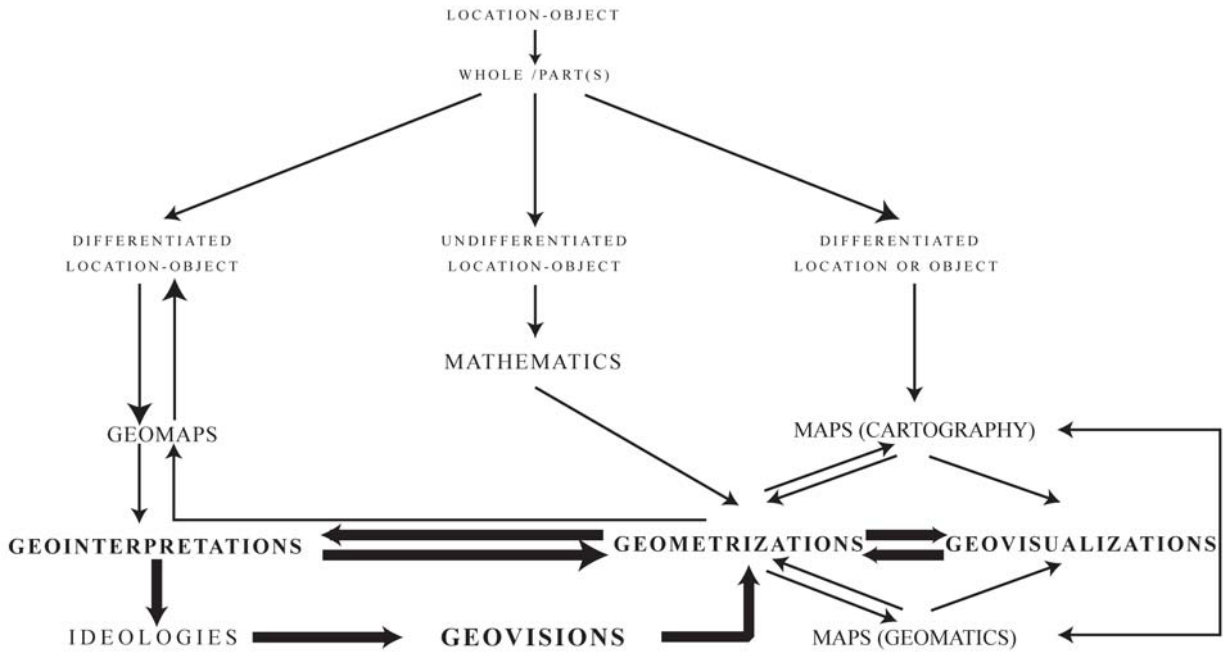
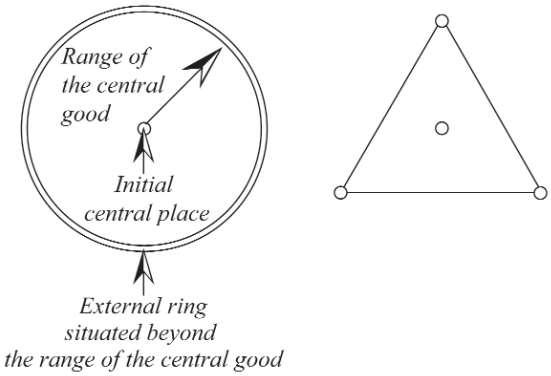


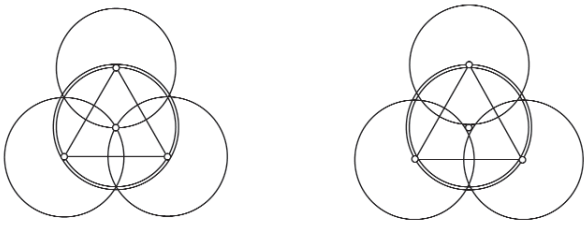

Figure 1: geometrization, geovisualisation and geovision. © Georges Nicolas, 2006




The problem of the distribution of the central good (Walter Christaller, 1933)

(Left) Problem stated by Walter Christaller in 1933 in "*Die zentralen Orte in Süddeutschland*": let be a "central good" having a "range" (20 km) proper to the "central place" from where it is distributed. How to distribute this "central good" in the ring (20-21 km) situated beyond the "range" of this good?

(Right) The solution stated by Walter Christaller, without geometrical demonstration, is that "it is necessary" to put three "central places" on the vertexes of an equilateral triangle, the initial place of which is the "center" (figure right).



(Left) First possibility : the new "central places" are arranged on the internal limit of the ring. One demonstrates mathematically (first error) that 1,4 % of the ring is not covered.

(Right) Second possibility : the new "central places" are arranged on the outside limit of the ring. One demonstrates mathematically (second error) that 4 % of the ring is not covered.

The geometrical solution proposed by Walter Christaller to the problem which he formulated is not "true" except for 1,4 % or 4%, or exact in 98,6 or 96 %. It is always mathematically false and thus is not a "model". It does not resolve the stated problem, independently of any empirical check.

Figure 2: Walter Christaller's geometrical errors

Adapted from M. Michalakis and G. Nicolas: "Le cadavre exquis de la centralité", 1986

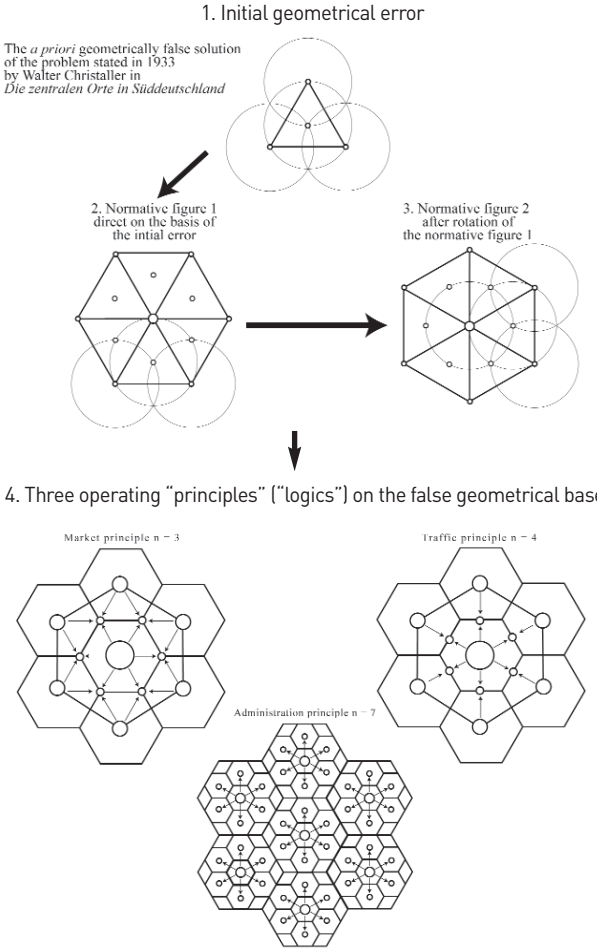


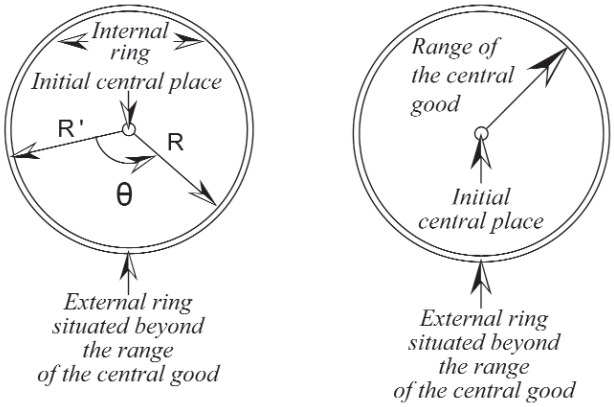

Figure 3: Walter Christaller : operating spatial systems deduced by means of figures built on a mathematically false base. © Georges Nicolas, 2006

hexagon, supplies six other "central places" at the vertexes of this hexagon. But each "central place", situated at the vertex of a hexagon also belongs to two adjacent hexagons. As a consequence, for Walter Christaller, the "central places" situated at the six vertexes of a hexagon are supplied—each for a one-third share—by three "central places" situated on three adjacent hexagons. For a full hexagon, the number attached to the "market principle" is therefore: 1 unit for the "central place" situated at the centre of the hexagon and 6 times one third for the "central places" situated at the vertexes, i.e.: n = (6 x ⅓) + 1 = 3.[1]

The second is the "transport principle". This is supposed to be the result of seeking for economy in transport between "central places". So as to reduce costs to a minimum, Walter Christaller suggests aligning secondary "central places" between the main "central places" along the diagonals which connect the centres of the initial hexagons. Each main "central place" at the centre of a hexagon supplies six "central places" situated on the sides which surround it. Conversely, each "central place" situated on one of the six sides of a hexagon is supplied for one half share by the two "central places" located on the adjacent hexagons on the side where it is located. For a full hexagon, the number attached to the "transport principle" is therefore: 1 unit for the "central place" situated at the centre of the hexagon and 6 times one half for the "central places" situated on the middle of the sides, i.e.: n = (6 x ½) + 1 = 4.

Third, is the "administrative principle". This is the result of a pyramidal spatial organisation of secondary "central places" around a main "central place". Walter Christaller situates the secondary "central places" at an equal distance from the main "central place" inside the hexagon. Each "central place" situated

[1] The letter k was introduced by August Lösch in 1940, in the first edition of *Die räumliche Ordnung der Wirtschaft*. Walter Christaller only used real integers that we are designating by the letter n in order to facilitate a comparison between his method and the one used by August Lösch.




Geometrical solution to Walter Christaller's problem (1933)

(Left) 1) The solution depends on the radius R of the internal ring, on the radius R' of the external ring, and on the angle θ between both radiuses.
2) Figures are built from any point situated in the internal ring.
3) All figures can make a rotation around the centre. They can have three, four, five or six sides.
4) The number of solutions is infinite. They have regular or irregular shape.
5) There is no normative shape.

(Right) Problem stated by Walter Christaller in 1933 in *Die zentralen Orte in Suddeutschland*. Let be a "central good" having a "range" (20 km) proper to the "central place" from where it is distributed. How to distribute this "central good" in the ring (20-21 km) situated beyond the "range" of this good?

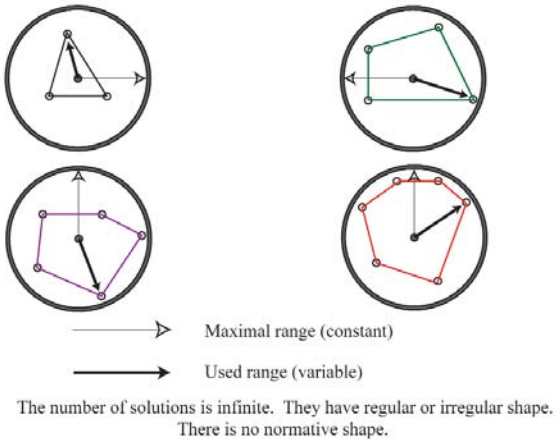



Figure 4: applications of the mathematically exact geometrical solution to Walter Christaller's problem: solutions with 3, 4, 5 or 6 edges. Adapted from M. Michalakis and G. Nicolas: "Le cadavre exquis de la centralité", 1986

at the centre of the main hexagon exerts its administrative and political power over six secondary "central places". For a full hexagon, the number attached to the "administrative principle" is therefore: 1 unit for the "central place" situated at the centre of the hexagon and 1 unit for each "central place" situated in the hexagon, i.e.: n = (6 x 1) + 1 = 7.

In Walter Christaller's theoretical diagrams, the circles are indeed equal to each other so that equilateral triangles can be constructed, to which regular hexagons can be correctly associated. But the numerical expression of these principles (the choice of which is very probably inspired by the "sacred" nature of the figures 3, 4 and 3 + 4 = 7 in the Judeo-Christian tradition) as they are derived from these diagrams, are no more than numerology i.e. using numbers in an attempt to foretell the future. The equation, which would allow these "principles" to be deduced from his triangular-hexagonal representation, is not formulated, nor is the necessary number of central places for them to function, mathematically justified.

That being so, far from attempting to prove mathematically how his geometric allegations correctly solve the problem he has raised, Walter Christaller generalises his invalid statements because, in his opinion, they are self-evident ("selbstverständlich möglich ")[2].To achieve this, he combines six equilateral triangles to form a regular hexagon on which he locates "central places", after which he interprets their location as "principles", valid in any space and at any time (figure 3). There is, in fact, an exact mathematical solution to the problem Walter Christaller submits (figure 4). It proves that the figures which solve the problem possess three characteristic properties 1) Their vertexes are not in the external ring formed by the extension of the "range" of their goods or services distributed beyond the maximum "range". They are in the internal ring between the minimum and the maximum ranges; 2) Apart from almost non-existent exceptions (one figure out of an infinite number of possible figures = 0 probability), the vertexes are not equidistant from the initial central place; 3) The possible theoretical range is not the range which is actually used (Michalakis and Nicolas, 1986).

## 2. A "THEORY" REFUTED

Walter Christaller's geometric diagrams cannot, therefore, be seen as a "model" since they do not solve the problem—which he himself submitted—of the location of the central places. And yet, he constructs his theory and attempts to verify it in Die zentralen Orte in Süddeutschland, by systematically using certain geometric properties of his mathematically unsound diagrams. The starting point is the measurement of the kilometric distances (as the crow flies) between Munich, placed in the "centre" and Prague, Vienna, Venice, Zurich, Stuttgart and Nuremberg. Walter Christaller draws six subsequent triangles: Stuttgart-Munich-Nuremberg, Nuremberg-Munich-Prague, Prague-Munich-Vienna, Vienna-Munich-Venice, Venice-Munich-Zurich, Zurich-Munich-Stuttgart. They are adjoined by their summit—Munich—and so form a polygon which is an irregular hexagon. He then isolates within this initial polygon the "German" part around Stuttgart with a boundary comprising Munich, Zurich (sic), Strasbourg (sic), Frankfurt and Nuremberg. These are towns with a population ranging from 400,000 to 700,000 inhabitants, of which two are not part of the initial polygon: Frankfurt and Strasbourg. He then measures the kilometric distance between the six towns (on average 261 km, with Munich-Stuttgart having a "normal" distance of 186 km), followed by the distance between towns with 20,000 to 30,000 inhabitants (some 72 km). In this way, he cuts out in the south "Germany" he had defined (including Strasbourg and Zurich), 18 shapeless "potato-like" areas with a "radius" of 36 km (36 x 2 = 72 km) and 59 "potatoes" with a

[2] *"Es schien überflüssig, die vorstehende Ergebnisse in Form von mathematischen Formeln auszudrücken; die mathematische Lösung ist selbstverständlich möglich und nicht schwierig"*: Christaller, Walter, 1933; p. 75. *"It seems unnecessary to express in mathematical formulas the results discussed in the previous paragraph. The possibility of mathematical expression is self-evident and is easily realized"*: Baskin, Carlisle W., 1966; p. 70.



"radius" of 21 km. Saving exceptions, around Munich and Nuremberg, the 21 km radius "potatoes" do not always intersect, whereas the triangular-hexagonal theoretical diagram postulates that they must all intersect. Finally, he calculates a "centrality index" on the basis of the number of telephones in all the political territories of Southern "Germany" (this time, leaving out Strasbourg and Zurich), so that he can classify areas with over 400 inhabitants in the following "central" hierarchy:

L: "Landeshauptstädte", "Länder" capital towns,
P: "Provinzialhauptorte", main towns in a Province,
G: "Gaubezirkshauptorte", main towns in a "Gau" (region),
B: "Bezirkshauptorte", main towns in a district,
K: "Kreisstädtchen", small (main) towns in a circle,
A: "Amtsstädtchen", small (main administrative) towns,
M: "Marktorte", market towns/places,
H: "hilfszentrale Orte", auxiliary central places.

This inductive-deductive approach (and not strictly deductive, as is often claimed by his followers) is guided and only functions thanks to its "ideal" triangular-hexagonal image of centrality. As a consequence, the six initial irregular triangles are taken as being equilateral triangles. They are then adjoined into a summit to form a hexagon, which should be regular. But, as Walter Christaller himself observes, the Vienna-Munich-Venice and the Venice-Munich-Zurich angles do not measure 60 degrees; instead they measure "nearly" 90 degrees and the angles in the four other triangles measure "nearly" 60 degrees, which corresponds to a circle of 420 degrees = (2 x 90) + (4 x 60)! From there, thanks to the geometric property of the regular triangle-hexagon figure that he uses as his basic diagram, Walter Christaller generates hexagons made up of equilateral triangles and, conversely, equilateral triangles composed of hexagons, using an extremely simple mathematical rule. He then calculates theoretically all the radii of his nested regular hexagons: 106, 60, 36, 21, 12, 7 km, as well as the radius of his basic triangle: 4 km, from the "ideal" theoretical distance of 186 km between Munich and Stuttgart. Consequently, the two distances on which he bases his reasoning are 21 and 36 km, obtained by using the erroneous geometric basis he postulates. On the one hand, he uses a basic distance observed only once: 186 km, and on the other, four angles (not six) measuring 60 degrees: Zurich-Munich-Stuttgart, Stuttgart-Munich-Nuremberg, Nuremberg-Munich-Prague, Prague-Munich-Vienna.

It is therefore hardly surprising to find that all his "real" numbers are always "approximate". For example, the theoretical distance of 186 km around Munich, whereas observed distances range from 150 to 360 km with an average (as the crow flies) of 258 km! Finally, as we shall see below, Walter Christaller manages to disregard numbers and figures when they very obviously invalidate his theoretical affirmations: at the very lowest level of his real hierarchy, he considers that the "normal" number of "central places" — M = 324 — can be used instead of the "approximate" number of "central places" — M and H: 180 + 192 = 372 — by eliminating the distinction between M and H, despite the fact that it appears in all the maps and tables representing his spatial results.

As a consequence, Walter Christaller's original theory is markedly different from subsequent more or less "revisited" reinterpretations. To sum it up, it contains three fundamental notions.

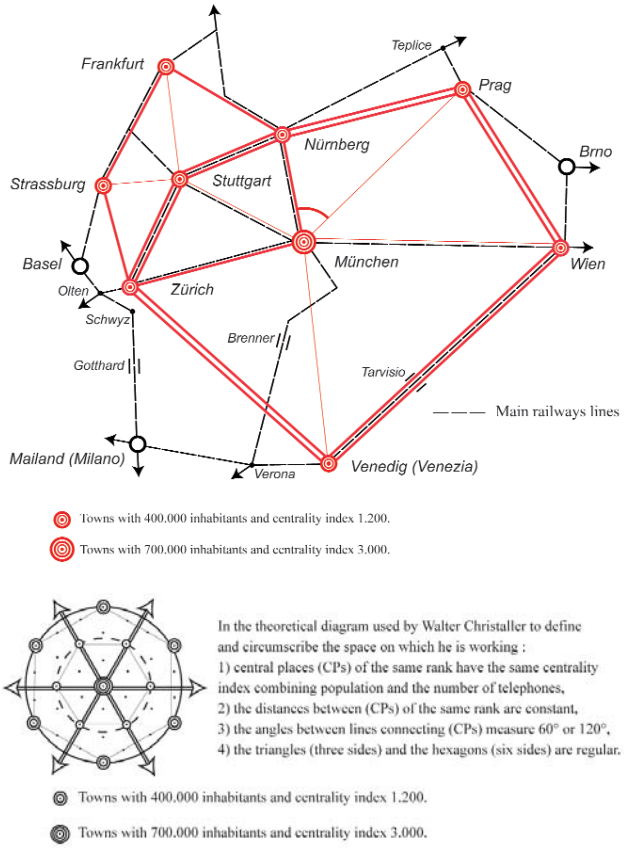
1) There exists in the world a total natural order, which is both organic and non-organic and is expressed in the form of an ideal spatial order that can be represented using triangular-hexagonal images, with which this order becomes comprehensible. The ideal total order ranks higher in rational terms than the real order, which is only too often no more than chaos that needs to be re-ordered, forcibly if necessary.

2) The position of places on the vertexes, the middle of the sides and inside the hexagons explains the fundamental principles governing the way in which the economy, society and its administrative functions operate. The task of places situated in these privileged positions is to concentrate production, consumption and administration activities and, as a consequence, people. They are central places serving as the foundation on which to organise space occupied by humans.

3) The human population is distributed discontinuously along the various stages of the hierarchy of central places, which is institutional by vocation. Central functions are distributed according to the hierarchical level of the places. Ordinary and elementary functions are to be found at all the central places, but at the higher levels of the hierarchy, functions become rarer and more specialised. There is therefore a constant numerical relationship between the distance separating the central places and the surface which they supply or administer on the one hand, and the population residing in these central places, on the other hand.

## 3. "MODEL" INVALIDATED, "THEORY" REFUTED AND "EXQUISITE CORPSES"

The obvious "naturalness" of these "central places" as "settlements" was restated categorically in 2005 in the Austrian research project ZORE ("Zentrale Orte und Raumentwicklung") by a joint (academia, federal government, regional government and townships) Working Group on a theoretical and applied revision of the "central place theory": "[...], central places have an eminent property: they represent "natural" central settlements and, due to the long term, countless shopping and location decisions made by private households as well by the public and private enterprises of the services sector, they have acquired their specific hierarchical ranks and "spatial acceptance" (Weichhart and al, 2005)." This faith in the validity of the "central places" theory (or to be more precise, in the "central places system"), despite its "incomplete and static (Pumain, Paquot and Kleinschmager, 2006)" nature and its multiple verification




Walter Christaller provides ranking for Straßburg, Prag, Wien, Venedig, Zürich, but does not give their population and centrality index values.

The "normal theoretical" distance between central places is 186 km. Real distances range from 107 to 360 km, with an average of 156 km around Stuttgart and 258 km around München. One single angle, defined by the basic triangle (München-Nürnberg-Prag), measures 60°. The ten other angles range from 40° to 85°. The irregular polygon around München has 6 sides, but the one around Stuttgart has 5 sides.

W. Christaller was unable to verify in "Southern Germany" that the "central places" were geographically situated according to his "principles". Walter Christaller's geometric diagrams cannot be seen as a "model" since they do not solve the problem which he himself submitted of the location of the central places.

Figure 5: Walter Christaller: Die zentralen Orte in Süddeutschland (1933): Construction of the South Germany Central Place System



deficiencies, is not the result of an "accretion" between new ideas and results with Walter Christaller's initial ideas and results (Brunet, 2000).

On the contrary, it is by neglecting or obliterating three quarters of a century's worth of contradiction between observation and theoretical postulates, by dint of erasing and censoring Die zentralen Orte in Süddeutschland, by moving away from or simplifying the ideal triangular-hexagonal "explanations", by unjustifiably bestowing diagrams by other authors upon Walter Christaller, by inversing the logic of the "central places system" and, finally, proposing contradictory geometric interpretations of its principles, that this so-called "theory" was salvaged.

We shall see how this "salvage" by successive amputations made it possible not just to rescue the only theoretical continuity that mattered, i.e. a certain notion of "order", but also how additions were grafted so as to keep alive what in fact was fast becoming a "scientific cadaver" as it progressively lost its original limbs. The process seemed to consist in adding new finery to an ageing collection of garments representing Christallerian centrality without the slightest regard for the old clothes that were being invalidated or discarded; a kind of "exquisite corpse" parlour game in which "a sentence, or a drawing is composed by several people without any of them being allowed to take into account earlier contributions (Breton and Eluard, 1938; Michalakis and Nicolas, 1986).

### 3.1. ASSERTION BUT NO DEMONSTRATION; DISSEMINATION OF UNSOUND RESULTS; PROCLAMATION THAT IDEALS ARE SUPERIOR TO REALITY

In fact, the initiator was Walter Christaller himself. Not only, as we have previously shown, did he massage his numerical results, considering them to be "almost" in conformity with his calculations even when they diverged significantly, but he also did not hesitate to oppose the "normal" property of his geometric diagrams (mathematically faulty) to the "real"—but theoretically unsatisfactory—characteristics of his empiric observations in Germany in the first quarter of the 20th century[3] (Christaller, 1933). Figure 5 is a reproduction of the "rational [theoretical] diagram of the central places" drawn by Walter Christaller top right on map n° 4 of his presentation of "The central places system in Southern Germany". It is, however, immediately apparent that the number of sides of his theoretical figure (six) does not correspond to the number of sides of his empirical figure (five) around Munich. And yet, in his detailed presentation of the various central place "systems" in Southern Germany, Walter Christaller wrote: "What is particularly remarkable and which strongly determines the structure of the Stuttgart L system, is the fact that here only five systems are contiguous and not six as is normally the case [sic = what is normally predicted by the theory]"[4] (Christaller, 1933).

In 1933, W. Christaller was therefore unable to verify in Southern Germany (including therein Strasbourg and Zurich!) that the "central places" were geographically situated according to his "principles". He could then: 1) allow that his diagrams were not operational, but refrain from suggesting an alternative: this was impossible since he did not know that his geometric model was mathematically unsound; 2) propose new diagrams without modifying his theory: this was also impossible since he believed that his diagrams were sound[5] (Christaller, 1933); 3) abandon his theory and his diagrams, formulate a new theory and construct another model, which never entered his mind[6] (Christaller, 1950).

[3] "The actual figure for occupation for each type of size [of central places] therefore corresponds particularly well to the normal diagram except for places G and A ... [concerns the Nuremberg "L system". [our italics]"

[4] "*Das zunächst Bemerkenswerte und das Gefüge des L-Systems Stuttgart in hohem Maße Bestimmende ist die Tatsache, daß hier nicht 6, wie normal, sondern nur 5 L-Systeme anstoßen.*"

[5] "Our diagram for the distribution and the size of the central places and the kinds of sizes is a rational one, meaning that it signifies the greatest degree of rationality in the economy, the best possible use of central installations and the smallest loss of "worth" (*Wert*). The economy is actuated through a principle of the greatest degree of rationality."

His reaction therefore was to assert that if reality did not conform to his theory, that was because reality was not "normal". He in fact participated in several attempts to modify reality forcibly by putting his ideas on land use at the service of Nazism and then Stalinist Communism (Rossler, 1988; Rossler, 1990; Rossler and Schleiermacher, 1993; Kegler, 2008).

Indeed, for Walter Christaller, "the theory has a validity completely independent of what reality looks like, but only by virtue of its logic and "the sense of adequacy" ("Sinnadäquanz"). [...] The unexplained facts must then be clarified by historical and geographical methods, because they involve personal, historical, and naturally conditioned resistance factors which cause deviations from theory" (Christaller, 1933). As a consequence, when he affirms but does not demonstrate, disseminates unsound results and proclaims the superiority of interpretable explanatory diagrams as a "model" for "reality", Walter Christaller paves the way for the "exquisite corpse of centrality" game, that is the dissociation of certain parts of the body of theory and the addition of new limbs without bothering to consider the consequences of previous dissociations.

### 3.2. AMPUTATE AND GRAFT TO KEEP DIAGRAMS ALIVE

And so, for Walter Christaller's direct followers, a theory in which one of the main components has been proved wrong by a description of reality, is still a valid theory and it is possible, using amputations or grafts, to use it to construct "models" which remain "useful", however scientifically unsound they may be[7] (Hagget, 1965).

The first opportunity for amputating is connected to the triangular-hexagonal diagrams, since for Walter Christaller, a positioning of places on vertexes, the middle of sides or inside the hexagons corresponds to an operating "principle" (figure 3). This method, consisting in deducing on a map the functions of places based on their theoretical geometric location, was disputed even during Walter Christaller's lifetime. For Hans Bobek (1927) and Maria Fesl (1978), certain "typically urban sectors of activity" ("typisch städtische Arbeitszweige": shops, finance, political and cultural professions) are apart from other economic activities (agriculture, mining, industrial production) and are concentrated at certain points ("Konzentration an gewissen Punkten") situated in the middle of the region they supply ("inmitten des von ihnen bedienten Gebietes"). Travel and relationship networks converge there, act like magnetic poles in the region and encourage the appearance of "urban centres". Since, for Hans Bobek, the degree of concentration of the economic activity of a region within a town decreases when distances increase, the result is that places take on a pyramidal or step-wise form of construction ("ein pyramiden—oder stufenförmiger Aufbau"), in which each larger than average central point is formed from several smaller central points (Bobek and Fesl, 1978). That being so, the rank of a central place can be evaluated on the basis of the number of central services it is host to, but not solely on the basis of the total population, nor even on the number of workers who live there. However, for Hans Bobek there is a close relationship between the rank of a central place and its population of consumers ("Größe des Bereiches": "the size of its range") wherever they may reside. Walter Christaller's theory according to which the "range" of a product is identical in all the "central places", regardless of their "level" is not verified, in fact, by observation: the goods produced by a Viennese baker have a greater "range" than those of a village baker. Therefore, according to Hans Bobek, the higher-ranking central places with a larger number of clients have a longer range than is the case for the same product in lower-ranking central places.

These observations invalidate Walter Christaller's triangular-hexagonal diagram, in which identical ranges are attributed to central places with different ranks. Subsequently, Hans Bobek published from 1961 onwards an Atlas of the Austrian Republic containing several maps of central places without using any geometric "model"(Bobek, 1961-1978)[8]. The hierarchy of the "centres" is constructed on the basis of the number of "clients" for each centre and not solely as a function of the population inhabiting them[9] (Bobek and Fesl, 1978). The "range" ("Bereich") is given by the set of consumers connected to a central place: whether these people live in the centre or more or less near to the centre is irrelevant. The role of "distance" in the calculation of the central place's rank is very minor. Hans Bobek replaces it with the notion of a "central rank" ("zentraler Rang") determined by the type of activity which goes on there and not by the type of spatial relationship ("Zentral als eine Eigenschaft bezieht sich für uns auf die Art der ausgeübten Aktivitäten, nicht auf die Art des räumlichen Bezugs.") (Bobek and Fesl, 1978).

The second dismissal of Walter Christaller's triangular-hexagonal diagrams were authored by the German spatial economist, August Lösch (1906-1945), who published the first version of his Die räumliche Ordnung der Wirtschaft in 1940 and a revised version in 1944 (Lösch, 1944). He died at the early age of 39 at the end of World War II, without ever having been a member of the Nazi party. August Lösch is said to have "generalised"

[6] While, after 1945, Walter Christaller dropped the idea of racial organic order, he remained focused on an "ideal order" for Europe, veiled by its borders, administrative boundaries and human population concentrations. He therefore suggests "that the disorder and what is opposed to order be made recognisable, so as to propose reordering and the creation of a new order [sic]. It will then become possible to approach an ideal of order, or ideal order, a task which must be undertaken urgently". To this end, he does not put forward natural components, but favours "the system of historic human and social central places which are distributed over the surface of the Earth according to precise rules and are integrated in a hierarchical system". He would like to reorganise the central places of Europe, in which he sets aside "real metropolises" ("*tatsächliche gegenwärtige Metropolen*"), the "true" geometric centres of countries ("*eigentliche Mittelpunkte*") and the ideal urban sites ("*Wunschbild-Metropolen*"). He criticises the actual location of Paris, London, Vienna and Berlin. He splits Switzerland into three systems with capitals in Paris, Rome and Berlin and suggests its capital be transferred from Berne to Lucerne.

[7] *"In his* Novum Organum, *Bacon describes scientific theory as consisting of "anticipations, rash and premature". Certainly we might argue that most of the models put forward [...] fit this description admirably; all are crude, all full of exceptions, all easier to refute than to defend. Why then, we must ask, do we bother to create models that study directly the "facts" of human geography? The answer lies in the inevitability, the economy, and the stimulation of model building. [...] In short the role of models in geography is to codify what has gone before and excite fresh [sic] inquiry."*
In this 1965 edition, Karl W. Popper's book, *The Logic of Scientific Discovery*, London, 1959, is listed in the bibliography. In the two-volume edition: Haggett, Peter, Cliff, Andrew D. and Frey, Allan, 1977, the reference to Karl W. Popper has disappeared. From that date onwards, refutation is no longer a spatial analysis method: as with Walter Christaller and August Lösch, the model is again superior to reality. The approach used tends once more to the "*rotten confirmation*" of the dominant mode of thinking and its ideology.

[8] Nor do the maps showing central places in the monumental *Atlas of Central and Eastern Europe* ([Jordan, Peter Pub., 1989 ss.], published later, contain the triangular-hexagonal diagrams . (Sauberer, Michael, Surd, Vasile and Tomasi, Elisabeth, 1990; Grimm, Frank-Dieter, Friedlein, Günter and Müller, Evelin, 1997).

[9] *"Zwischen dem Rang eines bereichsbildenden zentralen Ortes und der Gesamtzahl seiner Kundenbevölkerung [="Größe" des Bereiches) besteht eine enge Relation."*

Walter Christaller's central place system[10] (Hagget, 1965), of which he produces a separate theoretical interpretation and a significantly different graphic presentation, although he does use the same regular hexagonal shape. But in fact, August Lösch's diagrams interpreting the "principles" of the central places system, are often presented as being Walter Christaller's original diagrams, whereas they do not reproduce them and are neither in the same style nor drawn with the same graphic orientation[11] (Capel and Urteaga, 1982). Moreover, it was August Lösch who introduced the use of the letter "k" to describe the properties of the places on the hexagons, and not Walter Christaller[12]. In fact, unlike Hans Bobek, August Lösch radically challenges Walter Christaller's geometric and numerical flights of fancy from a theoretical—not empirical—standpoint[13] (Lösch, 1944). Unlike Walter Christaller, who claims to be working with deduction, but always starts off his theoretical considerations with empiric, and even aesthetic, observations (Christaller, 1933). August Lösch does not explain the function of a place in a region by its location on a triangular-hexagonal. He deduces the location of places within a hexagonal or square system using a system of theoretical equations, formulated a priori. These equations define the relationship between production or the capacity to distribute goods and products at each place, with the optimal distance for the distribution of these goods and products: "The distance between two enterprises of the same kind is equal to the distance between the settlements supplied times the square root of their number" (Lösch, 1944). It is not therefore the theoretical location which determines function, but the relationship between production/distribution and consumption which determines the optimal location. August Lösch seeks to demonstrate that the k=4 transport "principle" is axiomatically linked to the k=3 market "principle" and that the two cannot be separated as Walter Christaller did. He also demonstrates that the k=7 administrative "principle" cannot serve to administrate the whole of a complementary region if, as Walter Christaller does, the same orientation of hexagons in which are integrated the two other "principles", is retained[14]. August Lösch then goes on to prove that, in his concept (unlike Walter Christaller, mathematically demonstrated), each operating "principle" concerns a surface which is not the same as the surfaces of other principles, the shapes of which (hexagonal or quadratic) are much more numerous (about thirty) than the three identified by Walter Christaller. Finally, August Lösch shows that the regional distribution of the "central places" does not display the uniform pyramidal regularity claimed by Walter Christaller (in the k=3 system, the number of inferior dominated places is 2, in the k=4 system, it is 3, etc.). He demonstrates an irregular distribution based on variable density sectors[15] (Lösch, 1944). For Walter Christaller, the triangular-hexagonal figure is a given; for August Lösch, it is a result. Lösch also invalidates the equidistribution of complementary regions surrounding the central places. It is therefore false to claim that August Lösch "generalised" Walter Christaller, since their points of departure, their approaches and their results diverge significantly. The occasional use of the same geometric shape (a regular hexagon) is not sufficient evidence to eliminate such differences and divergences[16].

Despite this double invalidation in Walter Christaller's lifetime, the hexagon persists into the 21st century in the comments made by geographers, town and city planning specialists, historians, sociologists, etc. (Bailly, 1975; Vagaggini and Dematteis, 1976; Capel and Urteaga, 1982; Hagget, 1983; Ohji, Toshiaki, 1986; Lepetit, 1988; Pinchemel and Pinchemel, 1988; Kunow, 1988; Denzel, 1994; Staack, 1995; Short, 1996; Gilomen and Stercken, 2001; Lang, 2002; Vanagas, 2003; CERTU, 2001; Bathelt and Glückler, 2003). But the amputation and graft technique of the "exquisite corpse" becomes more complicated. In 1956, for example, in his M.A. thesis, the American geographer Brian Joe Lobley Berry (1956) begins by the statement that Walter Christaller's assertions on the location of central places in a (regular) hexagonal network are justified by a theorem.. that he does not set out! He claims, however, that this theorem can be formulated with the help of four "definitions" and three "axioms", all self-evident or beliefs. Definitions: 1) there are central places; 2) goods are distributed from these central places; 3) the space into which these central goods are distributed is a complementary region; 4) these goods are distributed and consumed by virtue of an economic behaviour. Axioms: 1) the price of the central goods varies according to the distance from the point of distribution; 2) there are internal and external limits

[10] *August Lösch recommends for reading Walter Christaller's "works on economic geography"* and praises his *"admirable book"*; Lösch, August, 1944, transl. Woglom, William H., 1954; p. 104, note 4 and p. 114, note 11.

[11] "k" is nowhere to be found in Walter Christaller's publications.

[12] For Walter Christaller the initial geometric figure is a triangle and not a regular hexagon. He starts off using the figure 2 to designate the two apexes of the triangle on which he situates the two lower places in relation to the third superior place which he situates on the third apex . (Christaller, Walter, 1933; p. 70; trad. BASKIN, Carlisle W., 1966; p. 65). On that basis, he deduces a geometric progression to explain how, in a system of complementary regions, the lower-order centres fit into the hierarchy compared to the superior centres, i.e.: a number of "complementary regions" equal to three in the "market principle", to four in the "transport principle" and seven in the "administrative principle". (Christaller, Walter, 1933; p. 72; transl. Baskin, Carlisle W., 1966; p. 67-68). In other words, for Walter Christaller, 3, 4 and 7 designate the number of places directly dominated in a hexagonal pyramidal hierarchy and not the numerical expression of a law permitting the number of places dominated to be deduced using a general equation expressing the relationships between places of production and distribution and the places of consumption, as is the case with August Lösch. (LÖSCH, August, 1944; p. 92, note 1; transl. Woglom, William H.,1954; p. 131-133, note 16).

[13] August Lösch considers that Walter Christaller's decision to choose hexagons in order to study *"the size and shape of [...] the [economic] region"* [...] as *"general though inadequate" (sic)*; Lösch, August, 1944, transl. Woglom, William H., 1954; p. 114 and p. 114, note 11.

[14] It is in fact this different orientation which makes it possible to identify and differentiate at first glance Walter Christaller's administration principle (Christaller, Walter, 1933; fig. 5, p. 83 and fig. 6 p. 84) and August Lösch's k=7 diagram (Lösch, August, 1944; fig. 36, p. 92).

[15] For a clear display of the differences between Walter Christaller and August Lösch in the construction of the hierarchies of places, see: Bathelt, Harald & Glückler, Johannes, 2003; fig. 38, p. 115.

[16] August Lösch did not generalise Walter Christaller: he reduced him to the status of minion in the service of a geographic "centrality" theory, apparently easier to understand and to teach than difficult systems of "spatial economy" equations. Compromised by his participation in the planning of deportations, exterminations and resettlements in the Eastern territories occupied by the IIIrd Reich, then by electing to join the Communist Party in West Germany after the second world war, Walter Christaller's interests were served by having it thought after 1945 that he had some scientific kinship with August Lösch. All the more so, because Lösch's refusal to join the Nazis attenuated Walter Christaller's proximity to them. August Lösch's good political reputation overshadowed and veiled Walter Christaller's trespasses. However, the absence of a sufficiently documented biography of August Lösch (Riegger, Roland Ed., 1971) makes it difficult to accept such proximity unless one's attitude is purely hagiographic (see for example: Haggett, Peter, 1965, p. 70-71). This is reinforced in economic terms, because August Lösch saw himself as "National-Socialist" in the meaning of the English economist Alfred Marshall when he referred to *"Economic Chivalry"* (Lösch, August, 1944; p. 258, note 2; trad. Woglom, William H., 1954; p. 364, note 2). August Lösch's hostility to John Edward Keynes (a disciple of Alfred Marshall), whom he considered to be a theorist of "chaos", never weakens throughout his *"Die räumliche Ordnung der Wirtschaft"* (Lösch, August, 1944; p. 177, note 3; p. 221, note 2; transl. Woglom, William H., 1954; p. 251, note 3; p. 308, note 81). August Lösch's affinities with National-Socialism were detected by: Derks, Hans, 1986; p. 258-9, notes 77 & 78; 2001; p. 177, note 75.



to this distance; 3) there is a relationship between the number of central goods and the population of the place from which they are distributed[17] (Berry, 1956). Brian Joe Lobley Berry then attempts to reconstruct the hexagonal image, using his "axioms" 1 and 2, that is the one with which Walter Christaller claims to explain the location of his central places based on the "provisioning (sic) principle" (k=3). Brian Joe Lobley Berry therefore produces an image which is supposed, he claims, to represent the spatial relationship between the "lower limit" and the "upper limit" of distribution of central goods (Berry, 1956). Unfortunately, he makes geometric mistakes and he fails to reconstruct Walter Christaller's original figure (figure 6). He is content with reproducing Walter Christaller's diagram, in a simplified and unexplained form, without using the complete hierarchy of signs for the central places (G, B, K, A and M) (Christaller, 1933; Berry, 1956). Nor does he provide a demonstration of his "centrality theorem" or, even less, of his reconstruction of Walter Christaller's hexagonal figure. If one does this work for him, using his figures 1 and 2 (Berry, 1956), the results do not tally with either Walter Christaller (figure 6) or with August Lösch (figure 7). It is therefore impossible to choose between Brian Joe Lobley Berry's two interpretations, since both are based on the use of an arbitrary numerical ratio (figure 8). Brian Joe Lobley Berry then refers to August Lösch's rotating hexagons, although he does not use them (figure 8), and so repeats another error: the mathematically inexact general equation attributed to the German spatial economist to calculate the number of "smallest [..] market areas"(Lösch, 1944)[18]. Brian Joe Lobley Berry nevertheless states that: "The rigid provisions of the Christallerian system, that these centers will have identical associations of functions and identical, unique population levels are relaxed" [sic] (Berry, 1956). In conclusion, Brian Joe Lobley Berry tries to reconcile mathematically the ratio between the population distribution of the towns in Walter Christaller's central places systems and the classification of the population of these towns in decreasing order by George Kingsley Zipf, according to a so-called "rank/size law"(Zipf, 1949; Robson, 1973)[19]. Once again, Brian Joe Lobley Berry commits a mathematical error and states a "law" which does not stand up to the test of theoretical verification (Berry, 1956)[20].

Following in the footsteps of August Lösch and Brian Joe Lobley Berry, a young German economist made an original attempt at amputation and graft of the "exquisite corpse" of centrality in 2004. After stating, with a truncated and out-of-context quotation from August Lösch, that "the advantages of a general geometric representation should be forgone"(Lösch, 1940), Dirk Fittkau actually uses a geometric figure to demonstrate that "coupled purchases" ("Kopplungskäufe") of at least two products in the same initial place of provisioning lead to the dislocation of the basic hexagonal system formed by an initial regular hexagon surrounded by six regular hexagons, all of the same size (Fittkau, 2004). In fact, for Dirk Fittkau, a coupled purchase in the initial central hexagon doubles the surface of its "market region" ("Marktgebiet") and transforms it into a "major supply places" ("großer Angebotsstandorte") which therefore covers partially the six hexagons of the "small supply places" ("kleine Angebotsstandorte"). Because of this, the six "small supply places" are incorporated into the sides of the initial central hexagon of the "major supply place" and their small hexagons disappear. In this way, we move on—although Fittkau does not say so—from the theoretical "market principle" (k=3) figure to the "transit (sic) principle" (k=4). But in fact, August Lösch does not challenge the use of the hexagons since he considers that they shed the light of geometric representation on to the generality of equations (Lösch, 1940). He does, however, criticise Walter Christaller for not using equations and only providing solutions based on "special cases", with as a consequence, that he deprives himself of the "advantages of general geometric representation" (Lösch, 1940). Which is precisely was Dirk Fittkau does when he presents figures without deducing them using equations defining their operating principles. But then, why does Dirk Fittkau use a geometric figure after having mistakenly and inappropriately attributed this objection to August Lösch (Fittkau, 2004)[21]?

Dirk Fittkau also refers to Walter Christaller and quotes him in a truncated excerpt: hexagonal images are only intended as the point of departure of "..the more realistic part of the theoretical reflection"(Christaller, 1933). But when Walter Christaller mentions the "factors" which make a central place important, he is not only referring to the creation of a "market region" ("Marktgebiet") triggered by the purchase of products as a function of supply in that place ("Angebotsstandort") as Dirk Fittkau is doing. On the contrary, Walter Christaller lists the numerous components which, according to him, limit the importance of the central place: complementary region, population, supply and demand of goods, conditions of transport, size of the central place, competition between a concentrated or dispersed mode of production of the goods. He then adds: "To deal with the interactive connections of these evolving components, we prefer to speak of processes ["Vorgänge"]—which are not, however, historic and concrete processes, but rather typical, "general" and abstracted from concrete and individual connotations, where time plays a role as an abstraction. These processes are closer to reality than purely static connections, they form the more realistic aspect of theoretical reflection and this part can be described as a dynamic theory"(Christaller, 1933)[22]. As

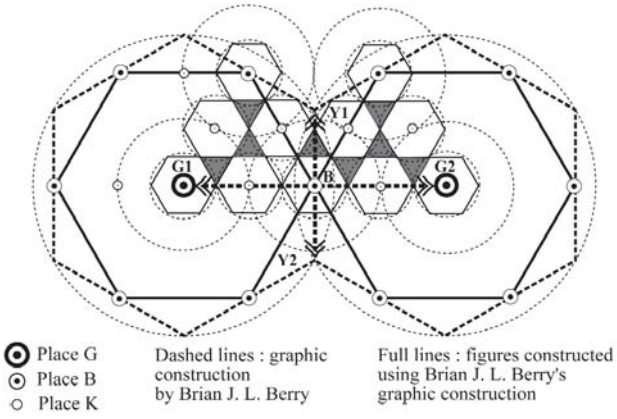
[17] In his 1967 publication (Berry, Brian Joe Lobley, 1967; transl. 1971), only the first "axiom" (price varies according to distance) remains (transl. p.111); the second "axiom" (there are internal and external limits to this distance) has been dropped (p. 110-114); the third "axiom" is simplified; the theory is only concerned with one "central and unique product", (p. 114-117) and no longer several goods distributed from a single central place. Only the hexagonal shape of the figures is retained, although it is impossible to understand how they are constructed using a single "axiom" in 1967 when three were needed in 1956.

[18] The equation Werner Känzig submits to William H. Woglom to calculate August Lösch's n "smallest possible market areas" is not: $(k\sqrt{3})^2 + 1^2 = n$ but: $(k\ a\ ^2/_2\ \sqrt{3})^2 + (i\ a\ ^1/_2)^2 = n$ with a: the distance separating the "original settlements" in abscissa: i a and in ordinate: In table 7, p. 119, the computations in the second column are invalid: the first result is 1 and not 7, etc.

[19] "*If our concern is with substantial aspects of cities, rather than with probability theory per se, the study of size distributions appears to be an elaborate maze which ends in a* "cul de sac".", p. 7; conclusion adopted by: Pumain, Denise, 1982; p. 70 and: Lepetit, Bernard, 1988; p. 178; Mandelbrot, Benoit, 1995: "I know of few endeavours [*Human behavior and the principle of least effort*] where so many strokes of genius, projected into so many directions, are lost in as thick a coating of weird fabrications, p. 180.

[20] Equation justifying the "rule" is given piecemeal and never assembled.

[21] "A consequence of changing from a one-product system to several products is that 'the advantages of a general geometric representation are lost' (Lösch 1940, p. 86). An economic picture painted by Lösch and Christaller's central places [...] are extremely fragile images [...]. They can only act as the starting point of "the more realistic part of theoretical reflection". (Christaller 1933, p. 86).




G1,G2: initial axis.

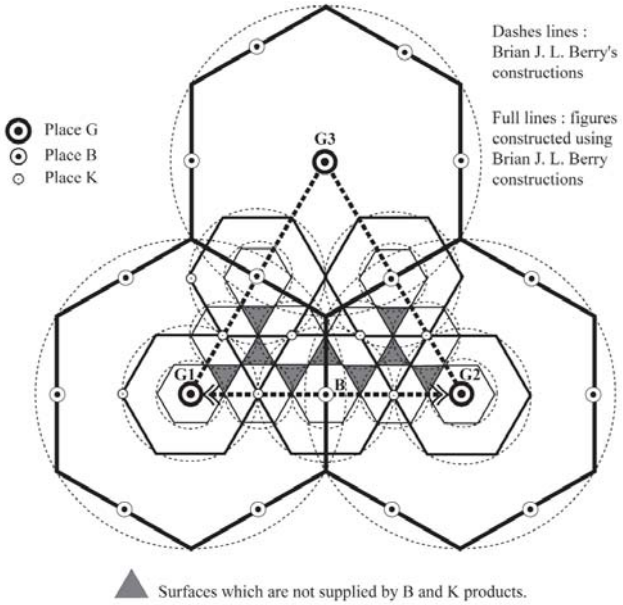
Two circles with identical radii and centres G1 and G2 represent the maximum range of the central goods G. Place B is arrived at by dividing G1,G2 by 2. The fundamental numeric ratio is 1/2: (G1,G2) x 1/2 = G1, B = B,G2.

The intersection of the circles of centres G1 and G2 defines the orthogonal axis Y1,Y2. Point B is situated in the middle of one of the sides of the construction hexagons resulting from the intersection of circles G1,G2. In the case of the "market principle" n = 3, the central places must be located (localized) at the vertexes of a hexagon and not in the middle of one side. A 30 degree rotation of the construction hexagons is needed therefore to get B on an apex and the size of the hexagons diminishes.

As the central place B is half-way between G1 et G2, similarly the central place K must be half-way between B and G1 or G 2 to respect the 1/2 ratio: (G1,B) x 1/2 = G1,K= K,B. The diameter of the central K hexagons is equal to half the G1,B distance. As the G hexagons do not have contiguous sides, K hexagons also have between them surfaces shown in grey, which are not supplied by K goods. The error originates in a confusion between the height (G1,B and G2,B) of the hexagon as a result of the intersection of the circles of centres G1 and G2 and the radius of the constructed hexagons. The numeric ratio used by J. L. Berry is 1/2 and not √3/2 between height and radius.

FIGURE 6: B. Berry's errors in his interpretation of W. Christaller's work in Geographic aspects of the size and arrangement of urban centers, 1956.






G1,G2: initial axis.

The G centre circles represent the maximum range for central products G.

Place B is arrived at by dividing G1,G2 by 2: (G1,G2) x 1/2 = G1,B = B,G2.

Point B is situated in the middle of one of the sides of the construction hexagons derived from the intersection of circles G1,G2.

Central places B are correctly located at the middle of the sides of the hexagons, following the "principle of communication" n = 4.

Central place K is situated half-way between B and G1 or G 2: (G1,B) x 1/2 = G1,K= K,B.

Hexagon B and K do not have contiguous sides. There are surfaces left between them (in grey for Ks) which are not supplied by B and K products.

The error originates in the confusion between the half-height of the initial hexagons and the radius of the constructed B hexagons. It reflects on the construction of the K hexagons.

B. Berry uses an arbitrary 1/2 numeric ratio instead of the mathematical ratio between the height and the √3/2 correctly used by W. Christaller and A. Lösch.

Figure 7: B. Berry's errors in his interpretation of Christaller's and Lösch's work in geographic aspects of the size and arrangement of urban centers, 1956



a result, not only does Dirk Fittkau neglect all the elements listed by Walter Christaller except for two of them: products (goods) and the region, but he also replaces the general "process" which links these elements and which cannot be "historic", nor "concrete", nor "individual", by an individual and concrete act of purchase ("coupled purchases": "Kopplungskäufe") in the presence of a multiple supply of products.

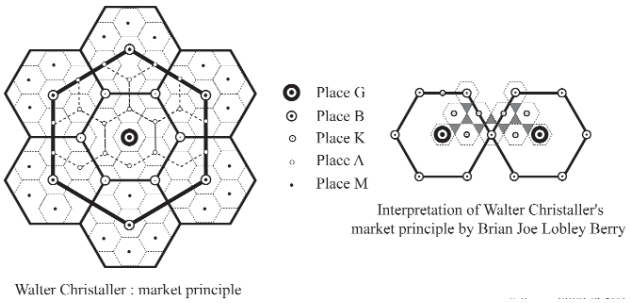
In the circumstances, "the more realistic aspect of the theoretical reflection" on the image of "[..] consequences on the size of the market [Marktgebiete] and of the advantages for the settlement [Agglomerationsvorteile]" takes on a very different meaning. We are no longer studying the connections between all the elements of a process, we are isolating two elements from the complete set of relationships (amputation). We then replace the non-historic, non-concrete and non-individual nature of the action by an individual, historic and concrete behaviour in the presence of a supply of products (transmutation). To complete the operation, all that remains to be done is to add a new element: the coupled purchase instead of the single purchase (graft). Dirk Fittkau is here defending the ideas put forward by his doctoral thesis supervisor, Jörg Güssefeldt (1941-2004), Professor of Economic Geography at Göttingen University, who was defending traditional German spatial economics under attack by the "New economic geography" referring to it under the name of "Germanic geometry"(Gussefeldt, 2003; 2005), rather than Walter Christaller and August Lösch's original ideas, which he cuts and distorts in an extremely original "exquisite corpse" process. In this latest version, there is amputation, transmutation and graft so that it becomes possible to do the exact opposite of what was initially announced: cease using a geometric image as a general representation and then use to transform it into the "more realistic part of the theoretical reflection".

[22] "Wir sprechen jedoch bei der gegenseitigen Beziehung sich verändernder Elemente wohl besser von Vorgängen—jedoch sind nicht historische konkrete Vorgänge, sondern von dem individuellen konkreten Verlauf abstrahierte „allgemeine", typische Vorgänge gemeint, wobei die Zeit als Abstraktum auftritt. Diese Vorgänge stehen der Wirklichkeit also näher als die rein statischen Beziehungen, sie machen den wirklicheren Teil der theoretischen Betrachtung aus, er sei als dynamische Theorie zusammengefasst."

### 3.3. CUTTING AND CENSORING

As soon as they were made public (Dörries, 1934; Bobek, 1935), W. Christaller's theories gave rise to comment in Germany and they were discussed at the 1938 International Geographical Congress in Amsterdam[23]. Later, in 1941, they reached the United States (Ullman, 1941). August Lösch's ideas were brought to the United States in 1938 following two visits he made in 1934/35 and 1936/37 (Lösch, 1938). After his death in 1945, he was described as "a man blessed" (Stolper,1954)[24] and a true anti-Nazi hero. He was translated into English and published in 1953. The translation of Walter Christaller's work (1893-1969) by Carlisle W. Baskin was only started in 1954 and published in 1966 (Baskin, 1966). Thirty-three years after the end of World War II, many of Walter Christaller's figures had become obsolete. It can therefore be argued that the cuts in the text (in particular in the numerical tables) are not an "impediment to understanding the work as a whole"(Robic, 2001). But it might also be considered that they bias Walter Christaller's original ideas since they were made after the reinterpretation of the "central places system" by August Lösch, who invented the use of the letter k to explain the "principles" and introduced a presentation of the hexagonal diagrams circumventing the use of equilateral triangles. Carlisle W. Baskin's cuts (36.5% of the text) bear on the preface ("Einleitung"), the detailed analysis of the central systems in Southern Germany ("Regionaler Teil"), except the one concerning Munich, most of the numerical data ("Tabellenwerk") and the original German bibliography, which was replaced by a bibliography in English in which Nazi-minded authors, or those whose position regarding the Nazis was ambiguous, were omitted[25].

The link between the cuts in the numerical data tables and the removal of the description of the south German "central systems": Nuremberg, Stuttgart, Strasbourg (sic) and Frankfurt, is obvious. Only the data concerning the Munich "system" were kept in the English translation, since they were the only part described in detail. But in fact, this "system" is the one for which the empirical data is the least disconnected from Walter Christaller's theoretical diagrams (figure 5). The disappearance of the introduction, however, introduces a serious discrepancy with the author's intentions, i.e. contribute through his research, to a "a new division of the German Reich" ("Neugliederung des Deutschen Reichs") (Robic, 2001): "The next part of the work was initially designed as a scientific exercise by the national economic State; the determining point would have been finding the theoretical economic foundations for a rational administrative State construction and a new division of the German Reich, and thus a simplification for the State [..]. Instead of the initial project, there was pure research concerning a more practical point: geographic and economic research on the law of regularity of numbers ("die Gesetzmäßigkeit") , of the [spatial] distribution and the size of urban places represented using the example of Southern Germany (Christaller, 1993)." These introductory remarks were dated in the summer of 1932, a few months before Adolf Hitler seized power (January-March 1933). Their removal in the translation after the war, in 1957-1966, paved the way for not mentioning Walter Christaller's intention to work on




B. Berry and W. Christaller: errors and distorsions

Eliminated by B. Berry:

1) reduction of economic activity to retail sales: agriculture and industry disappear;

2) simplification of the hierarchy: places A and M disappear;

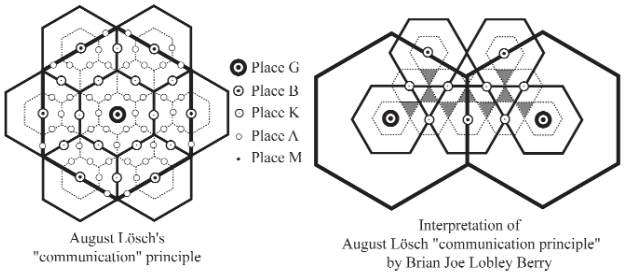
3) confusion between length of height and length of radii of hexagons: elimination of numeric ratio √3/2 to calculate distances between central places.

Added by B. Berry:

1) confusion between the length of the height and the length of the radii of the hexagons: use of the numeric ratio 1/2 to calculate variations in distances between centers;

2) "relaxing" of connections between population and economic activity.

B. Berry adds his own errors to Walter Christaller's initial error, in which his geometric diagram does not solve the problem since it is mathematically unsound.




Eliminated by B. Berry:

1) genesis of the hierarchy of the hexagons, starting with the smallest;

2) the hexagon rotation is dropped;

3) elimination of the numeric ratio a√n to calculate distances between central places.

Added by B. Berry:

1) statement that there is a "centrality theorem", which he does not demonstrate;

2) genesis of the hierarchy of hexagons starting with the largest;

3) statement that A. Lösch's "generalised" W. Christaller.

B. Berry adds his own errors to A. Lösch erroneous interpretations of W. Christaller's work.

Figure 8: B. Berry and August Lösch: errors and distorsions



[23] *Records of the International Geographical Congress in Amsterdam, 1938.* T. II, Section III a: Human geography (Chairman: Prof. A. Demangeon). July 21st Session. Question 2: Functional relationships between urban and rural settlements (Chairman: Prof. Albert Demangeon [Paris], Session Chairman: Prof. Charles Biermann [Lausanne], acting). Transcription and translation in: Djament, Géraldine and Covindassamy, Mandana, 2005.

[24] *"The economics of location, [...] exhibit the characteristics of a man blessed at the same time with originality and a sense of tradition and history."*

[25] In particular, his thesis supervisor Robert Gradmann (Gradmann, Robert, 1926 [not quoted by Walter Christaller]) and Werner Sombart (Sombart, Werner, 1930 [quoted by Walter Christaller]) who inspired his "deductive" method: Christaller, Walter, 1933; p. 16; transl. Baskin, Carlisle W., 1966; p. 4.

"creating a hybrid between economics and geography in an effort to rationalise the national territories (Robic, 2001), emphasising the "scientific" aspect of the project: a verification using an "[economic and geographic] law" of an [..] elementary form of the order of a common sense of belonging [..] inorganically and organically, in other words the arrangement of a mass around a nucleus, of a centre: a central order ("eine zentralistische Anordnung"). This order is not only a form of human thought which only exists in the world of human representation and born only of man's need for order; it also actually exists in laws internal to matter (Christaller, 1933).

The cut in the third part of the original in German (Regional Part: "regionaler Teil") is just as significant. This is a detailed description of the Stuttgart "central places system" that Walter Christaller describes as being "here, contiguous not to 6 L systems, but, as is normal [sic= normally postulated by the theory], only 5. (Christaller, 1933)[26]" He does not question his non-functioning diagrams, does not propose an alternative with new diagrams, does not modify his theory and does not offer a new one. All this is perfectly coherent since Walter Christaller considers that "Hence, the theory has a validity completely independent of what reality looks like, but only by virtue of its logic and the "sense of adequacy" ("Sinnadäquanz") (Christaller, 1933) [27]". In consequence, when results do not conform to reality, they are seen as "abnormal" and can be explained historically and geographically as "deviations (!) from theory (Christaller, 1933)[28]". The idea that a theory can be refuted and the diagrams (the "model") invalidated never enters the minds of Walter Christaller or of his followers: they consider that a theory is not invalidated, it is verified. "They [the diagrams] have nothing to do with the theory itself, and above all cannot be cited directly as proof against the validity of the theory" (Christaller, 1933)[29].

But for the purpose of research, this method is very practical and particularly effective institutionally. If the adequacy between the results of observation of the spatial relations between towns in Southern Germany and the theoretical diagram of the central places system (later described as the "model") which is supposed to explain it (the so-called theory), is disputed, then the "normal" response is that the model being rationally "ideal", anything which does not fit into it is simply a less rational "deviation". The "model" must therefore be used to re-arrange reality which thereby takes on a higher degree of rationality ("Das Prinzip höchster Rationalität" = "The principle of highest rationality") (Christaller, 1933), and also becomes more effective, even if it means manhandling reality, by force and violence, if necessary. If, conversely, attention is drawn to the force, which must be used to apply the "theory", it can be argued that the scientific legitimacy of the theory and the purity of the model are not to blame, but the use made of them. Practice justifies theory, and theory excuses practice[30].

The disconnection between diagrams, theory and results allows Walter Christaller to advocate deduction based on irrefutable "principles" while he is actually practising induction (Part I A: "Grundlegende Begriffe = Fundamental meanings"), after which he can give a "static" description of the "central places system", the geometric expression of which is in contradiction with those very principles (Part I B: "Beziehungen der Statik = Static relations". To complete this first part (Part I C: "Vorgänge der Dynamik = Dynamic processes"), he reconstructs "dynamically" his "central places systems" by massive recourse to the data that he had classified as not being pertinent for his principles (in particular the figures for the urban population). Then, in the second transition part ("Verbindender Teil") and particularly in the third ("Regionaler Teil"), he can reconcile results and principles since his theory has a "validity, which is completely independent of the appearance of reality". This is not a "hypothetically-deductive" method; it is "dogmatically-justificatory".

In this way, in the "principles" (Part I A: "Grundlegende Begriffe = Fundamental meanings"), Walter Christaller examines at length and in great detail which "principal characteristic" ("Hauptmerkmal") qualifies a place as "central". He recognises that there are dispersed inhabited

[26] *" ... die Tatsache, daß hier nicht 6, wie normal, sondern nur 5 L-Systeme anstoßen."* [L: "*Landeshauptstädte*", capital cities of the "*Land*"]. See also 199, 216, 234, 232, 233, 235 and 251.

[27] *"...die Theorie hat eine Gültigkeit ganz unabhängig davon, wie die konkrete Wirklichkeit aus sieht, nur kraft ihrer Logik und 'Sinnadäquanz'"*.transl. BASKIN, Carlisle W., 1966; p. 4-5.

[28] "Abweichungen von der Theorie...", transl. Baskin, Carlisle W., 1966; p. 5. Which is an illustration of the opinion Walter Christaller has of the work—based on classic erudition and description—done by his historian and geographer colleagues!

[29] *"... sie haben mit der Theorie selbst nichts zu tun und können vor allem auch nicht ohne weiteres als Beweis gegen die Richtigkeit der Theorie angeführt werden."* transl. Baskin, Carlisle W., 1966; p. 5.

[30] After acknowledging that "he was neither the first, nor the only one, nor the best of theorists working on the town considered as a centre of connections", Marie-Claire Robic adds her voice to the latest campaign for the rehabilitation of Walter Christaller, initiated by some American and German geographers. Since Walter Christaller was dealing with "administrative meshing issues and administrative planning", his theory of central places should be "re-examined or re-inserted" in his voluminous "scientific" production on the subject of administrative reform, before, during and after the Nazi regime (Preston, Richard E., 1992). The violence arising out of the implementation of Walter Christaller's ideas would not invalidate either the scientific legitimacy of his "theory", or the beauty and simplicity of his geometric "model" (Robic, Marie-Claire, 2001; p. 158). Walter Christaller's honesty appears as "evident" in the way in which Marie-Claire Robic dissects map 4 of *Die zentralen Orte in Süddeutschland*: she reproduces it cut into two parts, so that she can mask the five-sided irregular figure which is supposed to "verify" the six-sided regular hexagonal theoretical diagram that Walter Christaller did not reproduce on his own map (see figure 5). The two concentric theoretical circles, however, which are the basis for the regular hexagonal image, that Walter Christaller inserts top right on his map and that Marie-Claire Robic reproduces, are supposed to illustrate convincingly "the confrontation between theory and reality ("*Wirklichkeit*") in the distribution of places K and B around places G in "Southern Germany". But an examination of the half map published by Marie-Claire Robic shows that numerous places B are to be found on the place K circles and these latter are abundant on the place B circles. Furthermore, cutting out half of the original map enables Marie-Claire Robic to state that there are six "metropolitan" (capitals, provincial capitals?) regions around Stuttgart, although Walter Christaller only identified five (Robic, Marie-Claire, 2001; p. 164)! Now, if the Stuttgart "system does contain *six* central places, theoretically the sum of one "central" hexagon plus *six* "peripheral" hexagons adds up to seven, not six, regions. As a consequence, the rehabilitation of Walter Christaller's pretensions to re-arrangement, despite the criminal use to which he put them during the Second World War and his outrageous proposals to transfer European capitals after the conflict, is reason enough to forget his scientific approximations and errors, since the "normal" response to these lapses is that since the "model" is rationally "ideal", anything which does not fit into it is simply a lower-order deviation from rationality. So that Marie-Claire Robic can write "... [*Die zentralen Orte in Süddeutschland*] is supported by a stake in the rationality of the social order—governed in this case by the State—to which the author has radically (sic) and continuously contributed" (Robic, Marie-Claire, 2001; p. 188). The statement could not be bettered: an ambitious opportunist, desperately seeking academic integration, is presented as a "somewhat self-taught *outsider*" (Robic, Marie-Claire, 2001; p. 153), a champion of authoritarian State-led spatial order improvement, borne by any political order (totalitarian or liberal) as long as it is a "central order" ("*eine zentralistische Anordnung*"): Christaller, Walter, 1933; p. 21).

locations which are not "points in the middle" ("disperse Siedlungen [...] die nicht Mittelpunkte sind = dispersed places [...] which are not centers [Mittelpunkte, sic!]": 1) "places connected to the surface (or dependent on the surface" ("flächenhaft gebundene [Siedlungen] = areally-bound [settlements]"): agricultural activities whose location is determined by the nature of the land; 2) "places connected to a point (or dependent on a point)" ("punkthaft gebundene [Siedlungen] = point-bound [settlements]"): mines, ports, points of passage (bridges, highway tolls, customs) determined by their specific locations (Christaller, 1933); 3) places which are not connected to their location, nor to a "central point", "area" or "absolute point (" indifferente Siedlungen, die also weder an einen zentralen Punkt noch an die Fläche oder an einen absoluten Punkte gebunden sind): monasteries, homeworkers, suburban dwellers around big cities, recreational facilities; 4) itinerant salesmen (Christaller, 1933). Walter Christaller therefore broadens the definition of a "central place" as given by his thesis supervisor, Robert Gradmann: "Hauptberuf—oder auch Hauptmerkmal—der Stadt ist es, Mittelpunkt eines Gebietes zu sein" ("The chief profession—or characteristic—of a town is to be the center of a region") and achieves this by an inductive observation of non central places. Since these "dispersed" settlements can produce "central" goods and services, meaning that they may have "central functions", the determining factor to recognise a "central place", is the concentration in its midst of "chief professions [functions]" ("Hauptberufe") (Christaller, 1933) on the one hand and on the other, the (minimal) sum of the distances that must be covered to benefit from them or enjoy their services. But the "distance" between the "central place" and its "complementary region" combines the price of transport, insurance, storage and the advantages and disadvantages of transit. The "distance" is the monetary sum of all these factors (Christaller, 1933). It is not therefore linked exclusively to the numerical size of the population (Christaller, 1933), to the position of the "centre" in geometric terms (Christaller, 1933) and to the number of kilometres between the centre and the settlements of its "complementary region"(Christaller, 1933).

The "sense of adequacy" ("Sinnadäquanz") depends on its "logic", however, and not on the "appearance of reality", so that after having deprived of legitimacy the kilometric distance and the geometric position at point A of the first part, Walter Christaller goes on to use them at point B, so-called "static", to construct the operating "principles" of his "central places systems" (Christaller, 1933). He also asserts that it is unnecessary to provide a mathematical demonstration of his figures (Christaller, 1933), effectively protecting him from any serious theoretical verification for half a century since he had put his geometric figures outside the reach of calculation and verification (things which are intuitively self-evident need not be verified!). Finally, at point C, after denying any determinant role for them in identifying "central places", he makes extensive use of the urban population figures to explain the "dynamics" of the "central places systems".

### 3.4. UNIFYING THE "EXQUISITE CORPSES"

The distinction between "ideal" and "actual" rationality introduced by Walter Christaller, in agreement on this point with August Lösch[31], means that the theory, the "model" and the facts can all be manipulated independently to piece together and manufacture a considerable number of "exquisite corpses", while claiming their conformity with their founders. But even better, it is possible to cut off the limbs of various "corpses" and re-assemble them by "accretion" or "aggregation", and, going even further, sum them up to fabricate "indestructible corpses".

This was done in 1962 by the German economist Edwin von Böventer working on the writings of Johann Heinrich von Thünen, Walter Christaller and August Lösch (Thünen,1826-1875): "Lösch's system can be taken to describe the spatial distribution in the secondary sector; Christaller's system may be applied to the tertiary sector, Thünen's system to the primary sector. (Boventer, 1963)". But in fact, a comparison of Walter Christaller's original writings and Edwin von Böventer's statements reveals to what extent he manipulated them to make them compatible with the works of Johann Heinrich von Thünen and August Lösch.

In Edwin von Böventer's attempt at unification of the "centrality" approaches, August Lösch is the key person because he is supposed to have "generalised" the founders' work, i.e. that of Johann Heinrich von Thünen and Walter Christaller. In fact, August Lösch criticised them severely and introduced hypotheses in the research on centrality which became as many constraints pushing research decisively in a direction that was not the one the "founders" were pursuing since they partially destroyed their initial ideas (table 2).

| Text not translated | Page numbers in original | Number of pages not translated | Percentage of the text's 340 pages |
|---|---|---|---|
| Introduction ("Vorwort") | 3 | 1 | 0.3% |
| Regional parts ("Regionaler Teil") | 182-251 | 69 | 20.29% |
| Tables ("Tabellenwerk") | 275-325 | 50 | 14.7% |
| Bibliography and sources ("Literaturverzeichnis. Sonstige Quellen") | 327-331 | 4 | 1.17% |
| TOTAL | — | 124 | 36.5% |

Table 1. Cuts in Walter Christaller's: Die zentralen Orte in Süddeutschland by the translator Carlisle W. Baskin: Central places in southern Germany.

[31] For August Lösch, economic settlements have a "rational location" (*"vernünftiger Standort", "rational location"*) which for their order, is superior to their "actual location", (*"wirklicher Standort", "actual location"*): Lösch, August, 1944; p. 1; transl. Woglom, William H., 1954; p. 4.

| Edwin von Böventer: "Towards a united theory of spatial economic structure" (1963) | Walter Christaller: Die zentralen Orte in Süddeutschland (1933), Central places in southern Germany (1966) |
|---|---|
| "Both Christaller and Lösch start their analysis of the structure of the landscape with a homogeneous plain ... at each point of this plain, the amount and quality of the natural resources, the production functions, the population density, the consumer preferences and all other economic and non-economic factors are identical" p. 168 | "The region for which a central place is the center will be called the complementary region" p.21<br>"Let us suppose a region ... which has ... inhabitants uniformly distributed over the entire region, except for one small place at which the population tends to agglomerate." p. 28<br>"Let us suppose that the population is distributed unequally in the region ..." p 29<br>"In discussing the distribution of population in a region, we should not consider only whether the population is central or dispersed." p 32<br>" ...the connection between the importance of a central place and the characteristics of the complementary region [...] are ... the size of the area, the landscape [topography and visible landscape...] the means of transportation, [...] natural endowment [...] fertility of the soil and minerals [...]" p. 43 |
| "Both systems [Christaller's and Lösch's] have in common the hexagonal arrangement of the production sites and the sales areas of a particular commodity." p. 171 | "The central good may be offered at only one central place within the region, namely B ...the places which should regularly supply the unsupplied ... must lie in the center point of those triangles which are determinate by each group of three neighboring B-places." p 61-62 |
| " ... to discuss the most important economic principles which determine the spatial structure of an economy [ ...] it will be demonstrated that all the existing models of spatial economic structure e.g., Thünen, Christaller and Lösch, are special cases of such a framework. p. 163<br>"The aim ... is to arrive at certain general statements within unifying location framework ..." p 165<br>"... the marginal principles have to be supplemented by the total conditions of equilibrium." p. 165 | "The crystallization of mass around a nucleus is, in inorganic as well organic nature, an elementary form of order of things which belong together—a centralistic order. This order is not only a human mode of thinking, existing in the human world or imagination and developed because people demand order; it in fact exists out of inherent pattern of matter." p. 14<br>"We seek the causes of towns being large or small, because we believe that there is some ordering principle [...] that governs their distribution." p. 2<br>"It should be stressed that the theory offered here is not complete. We set forth only such relationships and developments as are of considerable importance for the clarification of the concrete questions asked here. Therefore, the theory is not developed strictly systematically, but rather pragmatically." p. 5 |

Table 2

Lösch has five "conditions" for general spatial equilibrium (Lösch, 1944). 1) Producers' search for optimal location determines the sitting of production points which are also points of consumption. While it can be considered that Johann Heinrich von Thünen does comply with that given, although he is mainly interested in production, Walter Christaller takes an opposite view: the advantages of central location determine the optimal type of activity. 2) Minimising the market area dimensions maximises entrepreneurial profits because it reduces transport costs. This condition leads to merging the "minimal range" and the "maximum range" of the "central commodity" according to Walter Christaller. This has two consequences: a) the geometric solution proposed by Walter Christaller for the "central commodity" problem is still mathematically unsound[32]; b) August Lösch's hexagon rotating mathematical solution does not make it possible to progress from one level of Walter Christaller's central

[32] See figures 2 and 4.



places system to another (figure 9)[33]. 3) To achieve equilibrium in the spatial distribution of production-consumption activities, producers' profits need to be zero: this theory is in contradiction with Robert Gradmann's ideas (adopted by Walter Christaller) which defines the central place as a concentration of "chief professions [functions]" ("Hauptberufe") (Christaller, 1933). August Lösch's theory is, for that matter, so unconvincing that Edwin von Böventer and, in his wake, Walter Isard (1960), replaced the notion of individual producer-consumers with regional groups of producer-consumers. In this way, they can use evaluations of equilibrium between regions instead of a general equilibrium (Paelinck, 1988)[34]. Johann Heinrich von Thünen, for whom the "State" is in "isolation", does not seek equilibrium. Nor is this a concern for Walter Christaller whose priority is uniformity of the political and administrative hierarchy (Preston, 1992). 4) The market belonging to the producing and consuming concerns, whose surface is supposed to be a known factor, is in fact completely supplied with all required goods. For Johann Heinrich von Thünen, on the contrary, there are spatial limits to the "isolated State", determined by the reduction in land returns as distance from the central town increases (Thünen, 1826-1975). For Walter Christaller, administrative and political boundaries can be modified, so that "inter-regional trade" is related to a market whose boundaries fluctuate by virtue of the "principle" of seeking out an "economically harmonious landscape" ("wirtschaftsharmonische Zwecklandschaft") (Christaller, 1933). 5) At the frontier between two markets for the same commodity, price differences are zero for all producers concerned by the production of that commodity. August Lösch indeed demonstrated that Johann Heinrich von Thünen's predictions regarding the order of succession starting from the size of the expected profit or return as a function of the distance from the "central town": general diversified farming, forestry, alternating and triennial crop rotation and pastoral farming, can be inverted up to the point where the differences in returns from these farming practices cross over. It then becomes possible to transfer a culture beyond this point and therefore to inverse the order of succession of the resulting crop circles (Lösch, 1944). As a consequence, the distance to the central place, to a town in particular, is not the only determining factor for the distribution of economic activities in a "complementary region". The advantages derived by access to means of production, by soil fertility, by production and market scales must be added (Lösch, 1944). This totally invalidates Walter Christaller's a priori geometric location approach and renders impossible any systematic use of von Thünen's circles to study the distribution of activities around a place described as "central", in or around a town.

The "exquisite corpse" method consists in putting together ideas considered to be "true", with ideas that are known to be false, in the belief that the true will cancel out the "false" and make them come "true". With this method, there is no need to bother examining the initial ideas with a critical eye in case they might be wrong. In point of fact, adding by "accretion" (Elmi and Babin, 1996)[35] new mathematical errors to an initial mathematical error does not render geometrically true Walter Christaller's initial geometric error. But perpetuating the view that this geometrization was objective and a generator of ideals has encouraged and consolidated ideological geo-interpretations based on a central hexagon representation, so that it has emerged as a "geovision" based on authority and utility.

The amputation and graft process has continued without interruption since the end of World War II, more or less intensively at various times in the various geographic linguistic areas concerned (Dutch, English, Estonian, French, German, Italian, Japanese, Russian, Spanish, Swedish, etc.). Its detailed history should be proportionate to the hundreds of publications to which these multiple occurrences gave rise, which is of course out of the question in the space of a single article (Nicolas, Radeff and Adam, S.D.). Nonetheless, simply limiting observation to the beginning of the 21st century, it is possible to identify persistent reminders, in the latest of the "exquisite corpses", continuing to dissociate theory, the "ideal model" and reality, be it empiric or historic.

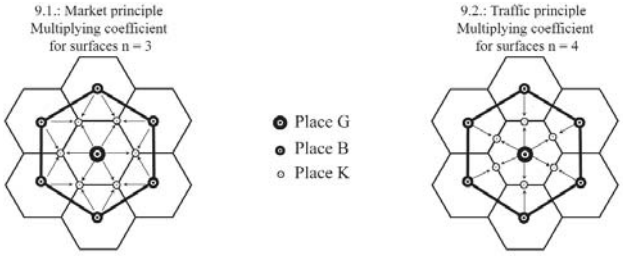
## 4. WALTER CHRISTALLER'S HEXAGONAL GEOVISUALIZATION OF "SPATIAL MARGINALITY"

The first reminder is the persistence of the triangular-hexagonal representation as the alleged tool for the integration of a demographic hierarchical concept as a "model" for a network of towns considered to be an "urban system". Justification for the use of this instrument of integration—despite the fact, according to its users, that it reverts to "an outdated era, because it is simplistic and geographic methods have evolved"—, is that it is "useful" for the purpose of reassessing an urban system and proposing possible or desirable spatial rearrangement scenarios (Woessner, 2008). For instance, when the construction of a connecting line for the TGV (high speed train) between the North-South main line (Paris-Lyon-Marseille) and the West-East main line (Paris-Strasbourg) in the middle and lower Doubs valley (an affluent of the Saône river which flows into the Rhône), one planner proposed the creation of a "Rhine-Rhône Metropolis", using this "Rhine-Rhône Corridor", in the form of a new kind of "complex system"(Woessner, 2008; Pumain, Paquot and Kleinschmager, 2006). His point of departure was a combination of Walter Christaller's and August Lösch's theoretical diagrams—despite the geometric misconception of the one and the

[33] The famous August Lösch figure: "Region with equal structure k = 4 ", wrongly attributed to Walter Christaller (Lösch, August, 1944; fig. 35, p. 92, transl. Woglom, William H., 1954; figure 35, p. 132), partially respects the 3, 9, 27 "rule of progression" for place dependence (Christaller, Walter, 1933; p. 72, transl. Baskin, Carlisle W., 1966; p. 66-68). There are in fact, for each place G, three dominated B places. However, if a line is drawn to join identical places K, putting them at the vertex of a hexagon, by virtue of the "market principle" n=3, the result is a figure in which hexagons of identical rank do not cover the entire surface (figure 9). Furthermore, at all hierarchical place levels, the figure has triangular "holes" between hexagons jointed by their summits and not by their sides. It is therefore impossible to pursue Christaller's numeric progression beyond 3 because August Lösch rejects a uniform distribution according to the size of the places: "[..] the same area will usually be the market for several goods, since there are more products than regional sizes. But beyond the market area these goods need have nothing in common". (Lösch, August, 1944; p. 85, transl. Woglom, William H., 1954; p. 122). In this case also, August Lösch did not "generalise" Walter Christaller. He brought him

[34] Using the "region" instead of the "place" paves the way for lavishing advice on "planning" and "arranging" on the basis of offers of financial compensation between regions. That being said, in the United States, once criteria for Federal grants became identical over the whole country, it was no longer necessary to prepare regional applications for grants based on comparative justification. The Regional Science Department of the University of Pennsylvania, founded in 1956, lost its status in 1993 (Davezies, Laurent, 2008; p. 41).

[35] Rather like crystals collecting into "ever-larger conglomerations" (p. 19) and forming rocks which, as they are destroyed by erosion, accumulate in basins, sink down and are cooked by the heat and melded so that they are reformed into new rocks. These new rocks, added to the older continental structures, build up new continents by "accretion" (p. 25).




Walter Christaller's principles (Die zentralen Orte in Süddeutschland, fig. 9.1 et 9.2)

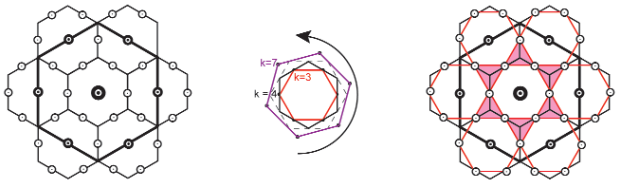



Figure 9.3.: For August Lösch the "communication principle" is "axiomatic" (*Die räumliche Ordnung der Wirtschaft*, p. 92, note 1). He therefore postulate n = 4 = k and draws the corresponding figure (fig. 9.3) which is not a reproduction of Walter Christaller's original figure (fig. 9.2).

Figure 9.4.: The different orientation of the hexagons is due to the "law" formulated by August Lösch to calculate the market areas (*Die räumliche Ordnung der Wirtschaft* p.81, fig. 27) with the smallest one (Walter Christaller's) as a starting point: n = 3 = k (fig. 9.4.).

To represent the geometric formulation of his "law" August Lösch draws his k = 3 first hexagon in an orientation similar to Walter Christaller's hexagon n = 3 (figure 9.1). August Lösch then uses the radius of the smallest hexagon (n = 3 = k, in red in fig. 9.4.) and equals it to the height of the next hexagon (n = 4 = k, in dark in fig. 9.4.). But in a regular hexagon: height = radius x √3 / 2, so that the radii of all successive increasing hexagons can be calculated and drawn with 30 degree rotations. The algorithm is:
radius 1 = height 2 → height 2 / (√3 / 2) = radius 2 → radius 2 = height 3 → height 3 / (√3 / 2) = radius 3 etc.

Figure 9.5.: August Lösch cannot use his theoretical succession, starting from the smallest n = 3 = k because, according to him, it is k = 4 which determines the whole system axiomatically. As a result, to deduce Walter Christaller's central place n = 3 (red hexagons in figure 9.5.) from August Lösch's central place k = 4 (dark hexagon in figure 9.5) and to rotate, the initial surface has to be reduced instead of increased.

"Holes" appear (pink triangles, figure 9.5.) in contradiction with the need that all "parts" of the system must be supplied with central goods or services (*Die zentralen Orte in Süddeutschland*, p. 69).

Figure 9: August Lösch did not "generalise" Walter Christaller

impossibility of using the hexagon rotation method of the other—with the aim of integrating the two diagrams (figures 2 and 3). This new representation of a central places system displays three large hexagons around three "central places" of the "1st rank", in the middle of which are pin-pointed three other and smaller hexagons around three "central places" of the 3rd rank, described as having "marginal positions" (figure 10).

This vision of "spatial marginality" is based on three Christallerian ideas: 1) all urban systems are organised around central places whose operating "principles" are determined by their position on a triangular-hexagonal diagram; 2) activity in the central places lead to a hierarchical concentration of functions and population: the more intense the activity, the larger the population; 3) around the central places, space is organised in a hierarchical set of nested triangles and hexagons. The author adds two of his own ideas: 1) the connections between "central places" of the 1st rank are privileged traffic "corridors"; 2) financial and economic globalisation generates a new separate hierarchy of "global cities" which combine with the older hierarchy of "central places" at all stages of spatial organisation. And yet, in practice, this design of "regional system geometry" (Woessner, 2008) integrates neither the empirical observations of north-east France's urban network nor those of the regions on the German and Swiss borders.

To begin with, the theoretical "Christaller revisited" diagram has three stages, whereas there are four in data on urban polarisation, six in the proposal for the creation of a "complex system" and 5 in the hierarchy used to define the "global cities (Woessner, 2008). So where would the "central places" of the "1st rank", which are supposed to be at the vertexes of the triangle formed by the "traffic corridors", be sited? If the regularity of the basic triangle is disregarded, the "global cities" Paris and Lyon are obvious candidates for two of the vertexes: but where should the third one be? The author is unsure and hesitates between Frankfurt, Strasbourg and Basle. Moreover, in all three cases, if the "global cities" are used, the main "corridor" which must coincide with one of the sides of the theoretical triangle, is in Germany (figure 10: Frankfurt) and in Switzerland (Basle, Lausanne, Geneva), but not in France where the "Rhine-Rhone Corridor" is supposed to be. Furthermore, supposing a "fuzzy summit" is adopted (Frankfurt? Strasbourg? Basle?) where is the level 3 centre at the junction of the three "cells in a marginal position" of the "revisited" diagram? The best-situated town is Dole (Dijon-Dole-Besançon), former capital of Franche-Comté deposed by Louis XIV who, after the second conquest of Franche-Comté (1674), moved the Parliament in 1676 and the University in 1691, to Besançon (Fietier, 1977). To make a show of modernism, Besançon would then be preferred (Dijon-Besançon-Belfort/Montbéliard), but in that case the "traffic corridor" would no longer be connecting the "central places" of the "1st rank" (Paris - Frankfurt? Straßburg? Basler? - Lyon) but instead "central places" of the "2nd rank", which contradicts the presentation of the theoretical diagram. Not to mention that the Rhine-Rhône Corridor would have one of its extremities chopped off: as it happens Mulhouse!

These inconsistencies are caused by the combination of two "logical" systems, that of traffic (k=4) represented by August Lösch's hexagon; and the "supply" (n=3) system represented using Walter Christaller's hexagon with the "revisited" diagram. It then becomes impossible to plot theoretic "corridors" (figure 10) between the central places situated at the vertexes of the hexagons functioning according to the "market principle" (n=3) passing also through the middle of the hexagons functioning according to the "traffic principle" (n=4). August Lösch had



actually understood this when, in his figure on "structurally equal regions" generated by the traffic logic (k=4), he dropped the idea of representing the central places based on the market logic (n=3). He simply stated that if all the places were situated in the middle of the sides of the hexagons, by surface [our italics] "each town dominates three other lower-ranking ones" (Lösch, 1944). August Lösch's theoretical diagrams cannot be coordinated with Walter Christaller's because their "systems" do not function in the same way and it is impossible to combine them to produce a new "model". In trying to "generalise" Walter Christaller, it is not even an "exquisite corpse" that is manufactured, but simply a "corpse" ripe for burial. Not only are these "revisits" unscientific, they are also useless, since all they do is generate confusion.

As regards the "Rhine-Rhône Corridor" and the Rhine-Rhône Metropolis", an alternative planning proposal to the one offered by Raymond Woessner could be formulated, based on the idea of a "metropolis" set in a "corridor", with sole reference to empirical observations, without having to bother with a geometrically erroneous "model". There is, in fact, a "potential axis" of traffic from Basle to Dijon, passing through Belfort-Montbéliard-Besançon between the Vosges and the Jura, crossing the south of a "basin territory" in which "regional towns" are situated. This "Rhine-Rhône axis" (CRR) would be connecting economic "competitiveness poles" in Alsace, Franche-Comté and Bourgogne with the "world-town" Basle on the Rhine in the north-east and the "world-town of Lyon on the Rhône in the south-west, and possibly generating a "Rhine-Rhône metropolis" (MRR) ) (Woessner, 2008).

## 5. PERSISTENT REMINDERS OF W. CHRISTALLER'S HEXAGONAL GEOVISUALIZATION

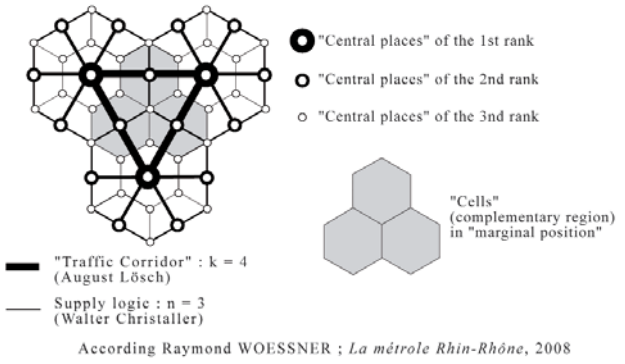
With some similarity to the previous reminder in 2002 of W. Christaller's work, a second occurrence is identifiable in a proposal to renovate the "central places concept" formulated by a Working Group of the "Akademie für Raumforschung und Landesplanung (ARL)" (Academy for Spatial Planning and Research) with a view to modifying the hierarchical classification of central places defined in Germany at the Federal level in 1968, 1970, 1972 and 1983 by the "Ministerkonferenz für Raumordnung (MKRO) - Ministerial Conference on Spatial Planning" (Blotevogel, 2002). The authors start with the statement that spatial planning is not to be confused with either spatial economics or with an empirical observation of central places systems (Blotevogel, 2002). After which, although the editor of the Working Group's conclusions is still convinced that there is such a thing as a "spatial model deductive of centrality" as formulated by Walter Christaller and generalised by August Lösch[36], the triangular-hexagonal representation is not used because, according to the members of the Working Group, the model is no longer appropriate for current geographic realities (Blotevogel, 2002). In point of fact, since the end of the 20th century, there are in Germany two kinds of "non central" settlements ("nicht zentrale Siedlungen"): 1) older inhabited places in rural areas, which have remained exclusively agricultural or are in the process of depopulation (Blotevogel, 2002) or settlements which are not included in the central hierarchy as defined by planners (Miosga, 2002; Heuwinkel, 2002): 2) new functional places: airports, high-speed transport nodes ("Hochgeschwindigkeitsverkehr-Knoten") large shopping centres and specialist retail complexes ("Selbstbedienung"—Warenhaus—und Fachmarktzentren" (Blotevogel, 2002), or else "clusters" in "sprawling urban regions" ("Stadtregionen "[sic] (Blotevogel, 2002) in metropolitan areas. Thus, at the beginning of the 21st century in Germany, there would be four kinds of spatial entities: ancient non urban spaces devoid of hierarchy or whose hierarchical order has disappeared; 1) regions in which the old central urban hierarchies still function after adapting to new economic and political circumstances; 2) regions in which old urban central hierarchies do not function satisfactorily; 3) new settlements integrated into the financial and economic globalisation systems whose non central hierarchies are more or less independent of the old central hierarchies.

That being so, the aim of spatial planning based on the renovated central place concept (CPC) ("Zentrale-Orte-Konzept (ZOK")), distinct from the centrality theory and empirical observation of the settlement systems, is to tidy up this central / non central confusion by proposing the implementation of a new hierarchy for central places in a re-unified Germany.

"Metropolregion GM" (Metropolitan region): settlement commanding supra-regional functions: services, finance, transport, science and research, culture and media;
"Oberzentrum OZ" (Higher-order centre): cluster of cultural, social and political activities with inter-regional relevance;
"Mittelzentrum MZ" (Intermediate-order centre"): cluster of economic and social activities to satisfy the needs of population at the regional level;
"Grundzentrum GZ" (Basic centre): cluster of services for the local population (Blotevogel, 2002).

In these circumstances, the possibility for the planners of promoting and managing such a Christallerian spatial "ideal" is not identical at all levels of the hierarchy, and all the more so because of Germany's political structure, i.e. with autonomous Länder, not centralised, which must be taken into account. At the metropolitan level of the Federal Republic and of the world, planners are limited in their action when they are proposing improvements to the transport system to facilitate the financial, economic and political command functions (Blotevogel, 2002). At the inter-regional level between the various Länder, however, there are more opportunities for action: improving work opportunities through effective management of means of transport (Blotevogel, 2002) with a reinforcement of coordination between regional centres to enhance the development of "intermediate towns" ("Zwischenstädte") between the different levels of urban hierarchy (Blotevogel, 2002). In the Länder, at the regional and local level, the planner's work is to coordinate the development of projects from one level of planning to the next

[36] " ... the hierarchy of central places is firmly retained as an image representing an ideal, that is so deeply ingrained that its foundations are considered indestructible. [...] In the presence of this flimsy evaluation of planning, the question arises of whether [geographers] are not deprived of some internal mechanism allowing them to abandon a path that has reached its limits and subscribe to a new paradigm."; Bathelt, Harald et Glückler, Johannes, 2003, p. 116.




Walter Christaller "revisited" by Raymond Woessner (2008)
© Georges Nicolas, 2008

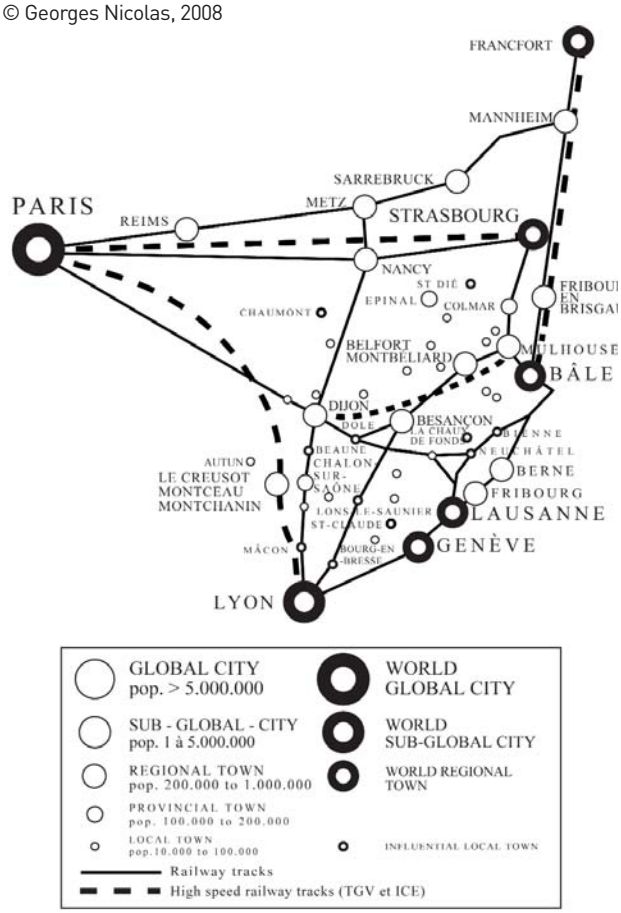



Figure 10: "Cells" (complementary region) in "marginal position": The last of Christallerian "Exquisite Corpse"?

According Raymond Woessner: La métrole Rhin-Rhône, 2008
© Georges Nicolas, 2008

("landesplanerische Zielvorgaben"), using "firm guidance" ("feste Rahmenvorgaben") (Blotevogel, 2002), derived from the renovated central places concept (ZOK). In this way, planners encourage the achievement of a consensus by managing competition between townships and moderating intrusion into projects by citizens, politicians, associations and private corporations and also guiding opinion in the direction of rationality and consensus building ("Rationalität und Konsensbildung"). For this purpose, the graphic representations of "geovisions" inspired by the classic outlines of the central places systems and the generalisations to which they gave rise are, according to the Working Group, interesting instruments for convincing and persuading because they are well known and generally accepted (Blotevogel, 2002).

Finally, although physically absent from the proposal for a renovated "central places concept" (ZOK) the authors wished to provide a convincing graphic representation of, the content of the Christallerian triangular-hexagonal imagery re-emerges and is reminiscent of the "ideal" hierarchical order in the command structures which is the irrepressible hard core of the exquisite corpses of centrality. This "ideal" image is so embedded in certain geovisions that authors use it, without even taking the trouble of presenting it graphically, to express explanatory "principles" justified with the help of a similarity of forms, even though these "principles" are contradictory by the very fact that they are based on a superior "order principle".

Thus, in 2006, in the article entitled "Theory of central places" in the Dictionnaire [de] la ville et [de] l'urbain, the triangular-hexagonal diagram is first mentioned to justify the "principle" of a theory formulated in France at the beginning of the 19th century which is supposed to explain "the number, [..] the size and [..] the spacing of towns" (Pumain, Paquot and Kleinschmager, 2006). In fact, in the article called "Town" in the Encyclopédie nouvelle (Reynaud, 1841), Jean Ernest Reynaud (1806-1863), a mining engineer , a graduate of the prestigious École Polytechnique and a philosopher who was a follower of the Saint-Simonian movement in the first quarter of the 19th century, but left it after 1830, asserted that peasants use land according to the physical status of the soil, water resources and cluster together by virtue of the "divine need to be sociable". When they settle in a circular area, the centre of which coincides with the site of their village, they reduce the distances they need to travel to till their fields. As neighbours in nearby villages do likewise, all these circles overlap and generate, by geometric simplification, regular hexagons. The organisation of the countryside is therefore the foundation of a spatial organisation which combines "order" and geometry and works in favour of conciliating reason and the historic legacies of religious faith. In consequence, according to Jean-Ernest Reynaud, "since the land is divided into rural hexagons", the "position of towns" can be allocated "by new hexagons embracing a certain number of the first hexagons, where the towns would occupy the centre" (Reynaud, 1841). He does underline, however, that this perfect hexagonal arrangement can only be verified if the territory on which its effect are felt is "uniform", which does not take into account the "anomalies" caused by the surface of the earth's "superficial inequalities" (Reynaud, 1841). But these inequalities are such that in the case of France, the result is that "in its natural borders, separated from continental Europe by the Alps and the Rhine, the centre of area, moving North, falls into a circle enclosed by Fontainebleau, Auxerre and Orleans. In this new French geography, the current territorial eccentricity of Paris is corrected. Antwerp (sic) compensates Marseilles; and the French

capital, balanced between these two ports, as close as possible to both, reconciles the recommendations of history and the demands of geometry, while keeping as much as possible to its present-day position. Carried away by geometric and patriotic sentiments, Jean Ernest Reynaud waxes lyrical in his conclusion and says: "To put it even better, there is already in France only one single city, and that city is France itself. Nature chose to situate this country in the fairest region on earth, in a place which is salubrious, fertile, commodious and varied." [..] "Its provinces are the city's districts; the fields and forests its gardens and walks; its rivers are its aqueducts; its highways are its roads; the capital is its forum (Reynaud, 1841)."

As there are no figures to provide the "number, [..] the size and [..] the spacing of towns" in Jean-Ernest Reynaud's work, it is only because the hexagon is used in both cases that the authors of the Dictionnaire [de] la ville et [de] l'urbain make the connection to Walter Christaller. They explain that in the "geographic theory" of centrality: "While the client populations [i.e. the centres proposing goods and services] are evenly distributed in space, the areas of influence take on the form of nested hexagons (Pumain, Paquot, Kleinschmager, 2006)". This reference to the geometric and geographic visualisation[37] appears to them as sufficient to justify a statement, making use of August Lösch's attempt at re-interpretation, to the effect that the "principles" for the distribution of the centres on these "nested" hexagonal figures explain the effects of centrality (n=3: market principle; n=4: transport principle; n=7: administrative principle: figure 3). This is a particularly flagrant example of the amputation + graft mechanism used to fabricate the last avatar of the "exquisite corpse" of the "theory of central places".

1) Amputations: 1.1) Not mentioning that an equilateral triangle is what enables Walter Christaller to construct the regular hexagon of the triangular-hexagonal figures. 1.2) Not mentioning that the mathematical solution proposed by Walter Christaller, to solve the problem he submits regarding the base of the equilateral triangle, is geometrically unsound; with as a corollary that, theoretically, the central places have a probability close to zero of falling into regular nested hexagons. 1.3) Omit saying that August Lösch's attempt to generalise Walter Christaller was a failure because it is partly mathematically erroneous and, above all, because the method of rotating the hexagons does not allow the deduction of Walter Christaller's "market principle" from August Lösch's "axiomatic communication principle" (figure 9). So that it can be safely stated that the ratio between the surface of the hexagons, the number of places concerned and the population supplied, is simply a progression of the number of "clients" related to a rise in the hierarchy of centres: "As regards the market principle, the client population of a centre is 3 times greater than the one of a centre of the level immediately beneath; this ratio is equal to 4 in the case of the implementation of the transport principle and 7 for the administrative principle". But here again, it is proven that this theoretical statement is mathematically unsound (figure 9). 1.4) Disappointed by the mismatch between Walter Christaller's central places system and observation, some researchers simply swept the corpse of the geometric "model" under the carpet, but did not give the reasons why they did so, and were not inspired to also exorcise the triangle-hexagon image. Bernard Lepetit was a case in point. Together with Peter Clark, he edited in 1996 the presentations made at an international conference on the history of economics which had been held in 1990, on the subject of capital cities and their "Hinterland" in modern Europe. While the notion of "hierarchical centre" is to be found, Walter Christaller is never mentioned (Clark and Lepetit, 1996).

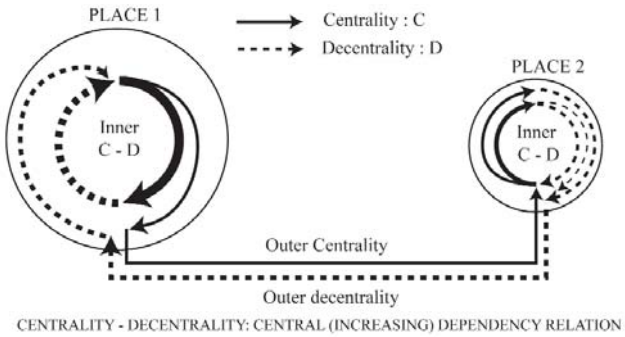
2) Grafts: 2.1) Only the results of empirical observations which can be interpreted as "proof" of the hexagonal theoretical geo-vision are mentioned; empirical or historical results which contradict the so-called "theory" are excluded from the theoretic formulation, even when they are recognised to be valid. In other words, when in current urbanised spaces, over half of the movements of consumers of goods and services are not directed towards the nearest centre to obtain a specific commodity, this counter-proof does not overturn the validity of the notion of theoretical "range" for each commodity (one commodity = one range), although this is a fundamental theoretical postulate of the "standard" central places system[38]. Furthermore, the proliferation of multiple-activity centres (such as supermarkets) invalidating the "market principle", totally annihilating the "transport principle" and introducing a distortion in the place hierarchy, is also unable to undermine a theory which claims to be spatially and temporally universal (Pumain, Paquot and Kleinschmager, 2006). Historians are therefore invited to seek further and further into the past a confirmation of a theory which was invalidated successively in the present, in modern times (Lepetit, 1988; Favier, 1993), in the Middle Ages (Fray, 2006) and in antiquity (Burghardt, 1979). Archaeologists and anthropologists are required to enter the fray, since the theory could be used to understand "nomad societies" and the "periodic market" systems despite their lack or scarcity of towns (Pumain, Paquot and Kleinschmager, 2006)! Nor must we forget the protohistorians who are supposed to have explained the origins of the Oppida by some supposed (but not proven) statistical regularity and shown their continuity with the towns of great empires such as those of the Roman Empire (Pumain and Van Der Leeuw, 1998). 2.2) The concepts developed by Jean-Ernest Reynaud and Walter Christaller are merged, although the former bases his hierarchy first on agricultural inhabited settlements and later on towns practising trade or exercising administrative activities

[37] *Rather, the problem with the German tradition must surely have been that it seemed to be about geometry, not about economics as the increasingly dominant Anglo-Saxon mainstream understood it"*: Krugman, Paul, 1995, p. 39.

[38] Despite the fact that Walter Christaller, who does not mind contradicting himself, says: "The same good has a different range at every central place ...": Christaller, Walter, 1933; p. 58; transl. Baskin, Carlisle W., 1966; p. 53.

("bottom - top"), and the latter bases his reasoning mainly on towns (Christaller, 1933) and deduces his hierarchical system from "top - bottom" (Christaller, 1933).

It is therefore clearly the persistence of the ideal hexagonal image which is guiding the fabrication of an "exquisite corpse", such as the one proposed by the Dictionnaire, associating partially contradictory geo-interpretations with the assistance of an archetype of the central spatial order. According to its authors, it is possible to merge Jean-Ernest Reynaud's community order and Walter Christaller's totalitarian order because they both express—as do other types of urban orders—mankind's need to be organised on the surface of the earth around a fixed point: the "Centre"(Pumain, Paquot and Kleinschmager, 2006). Thus, in the traditions of antiquity, for Euclid (-450, -380) "The earth is in the middle of the universe and plays the role of centre (Greek: "kentron") of the universe." (Aujac, 1993) For Plato (-428, -348): "The founder of a city must first establish it as close as possible to the centre of the country [..] after which, he will mark out twelve parts, reserving first of all an enclosure for Hestia, Zeus and Athena, which he will name 'acropolis' and surround with a boundary, and from which starting point he will divide the city itself and all the territory into twelve parts [..] Everyone shall have two dwellings, one close to the centre and the other at the extremities (Platon, 1975)." Similarly, in the "primordial traditions" of archaic societies, the sacred, infinite and transcendent, are dialectically united to the profane, finite and ordinary, in a non homogenous natural space in which paths range from one region of the cosmic being to the other (Relieu, 1992). The fact that this spatial order is currently in the throes of "decentralisation" in the form of centres springing up at the periphery of ancient historic nodes ("polycentrality"); the creation of new urban entities deprived of centres ("new towns"); the merging of old centres ("super-centres" or "hyper-centres"); the setting up of networks of spatial entities straddling areas which are sometimes very far apart, etc. would not modify the desire or plans to assemble around "mixed centres", "combining commercial, medical and health activities as well as sports, leisure, culture and recreation (Pumain, Paquot and Kleinschmager, 2006) so as to restructure tentacular urban entities ("sprawl cities") whose successive centres have been deserted by numerous activities, in particular industrial. In these circumstances, "urban celebration" would no longer be limited to the pleasure felt by Walter Christaller contemplating the "picture of a medieval town (Christaller, 1933)"; it would extend gradually back to the origins of towns where "centrality" is obvious and intact.




CENTRALITY - DECENTRALITY: CENTRAL (INCREASING) DEPENDENCY RELATION

The inner dependency of a dominant location-object is the result of its capacity to supply the products and services essential for the subsistence of its population as well as the means required for its social and cultural existence. The products and means in excess of these needs give it a capacity to put into central dependency (outer centrality) other location-objects.

The inner centrality of a location-object in central dependency is a measure of its capacity to supply all the products and services essential for the subsistence of its population as well as all the means required for its social an cultural existence.

Outer centrality-decentrality relations between dominant-dominated location-objects are not symmetrical. In exchange for products, services and social-cultural means made available to it (outer centrality), the dominated location-object supplies to the dominant location-object (outer centrality) a supplement of political, economic, demographic, cultural, institutional and monumental inner decentral capacity, which feeds and reinforces its centrality.

Outer decentrality, therefore, transmitted by place 2 to place 1, reinforces its decentrality and generates supplementary outer centrality. Inversely, in place 2, the transfer of outer centrality triggers a reduction of centrality and then of, inner decentrality together with supplementary transfer of outer decentrality in the direction of place 1.

Place 1 is central, place 2 is in the central dependency of place 1

Figure 11: Centrality - decentrality: central (increasing) dependency relation



This "patrimonial" historical geovision of the relationships between human settlements and their environment is invalidated by historical research showing that they have always been the scene of hostility between antagonist "central" and "decentral" forces in their midst (figure 11) (Nicolas, and Radeff, 2002). The problem actually arises at the outset in the following terms: what is the determining factor in the dialectic relationship between the sacred and the secular? The fact that where mankind gathers together is regarded as sacred or that economic, social, political, environmental and historical circumstances determined the choice and genesis of the place concerned? Furthermore, evolving criteria for "centrality" or their eradication show that an approach by the sole persistence of the ideal hexagon image, expression of a pyramidical hierarchization, does not allow a full understanding of the problem (Fray, 2006). While the internal centrality of a location-object is its capacity to supply to the population living there the products and services needed for their subsistence as well as the means which are essential for its social and cultural existence", the possible surpluses that this internal capacity can deliver determine the external centrality of the place-object, i.e. its "capacity to collect in the same place an offer of goods and services for external sale" (Pumain, Paquot and Kleinschmager, 2006). Use of this surplus enables the first location-object, using its external central capacity, to create a link of central dependence (external centrality) with a second dominated location-object. This latter location-object does not fully control its own economic, social and administrative existence, since it must transfer some part of it to the dominant



central location-object on which it depends (internal decentrality). Conversely, the dominant central place reinforces its internal decentrality thanks to these transfers and therefore enjoys a supplement of external central capacity of goods, services and possibilities "to sell them (supply or exercise) to the outside world".

These relationships between central-decentral location-objects do, in fact, work both ways, but are not symmetrical contrary to what is suggested by the hierarchical hexagon image. This lack of symmetry is paradoxically illustrated by the recent normative hexagonal imagery explaining the way in which Walter Christaller's "principles" function. In some cases, movement is inward, from periphery to centre, from the bottom to the top of the hierarchy ("bottom-top") (Short, 1996); whereas in others, movement is outward, from the centre to the periphery, from top to bottom of the hierarchy ("top-bottom") (Pumain, 2004). This truncated and unilateral approach in describing the centrality-decentrality relationships makes it particularly difficult to arrive at a historical and geographical differentiation of the location-object "borough", "town", "metropolis" etc. if only the classic hexagonal image of the "central places system" is used as the archetypal emblem of a so-called "theory of centrality". As a result, the uncorrected or forgotten errors, the approximations to the truth accepted to the degree that false affirmations are stated to be "obvious" foundations, are ratified by the reintroduction of a transcending irrational dimension to oppose immanent rational understanding in the "theory of central places, revisited" of the spatial entities of human settlements. But this so-called "theory" survives by using a self-justifying remnant hexagon imagery: the ideal image guides the exploration of reality and only those aspects of reality which support the ideal image are validated. The geo-interpretation of "centrality" determined by the a priori choice of a projection system by the observer, on the one hand, and by his beliefs or ideology expressed through an explicit or implicit hexagon geovision, on the other hand, determines the use made of the results of observation and that of the representation of the central-decentral location-objects.

## 6. THE DIALECTIC OF FORMS IN "GEOGRAPHIC VISUALIZATION"

External reality as an object precedes the approach by the geographer using the differences in reality to acquire knowledge of it. Geography cannot exist without the Earth, which is its original object. On Earth, all objects have a place, but it is impossible to determine a priori if an object is, or is not, a geographical one. As a consequence, any terrestrial location-object is first of all a spatial entity belonging simultaneously to two sets: the locations set and the objects set, and each information concerns two elements forming an indissociable pair: a location and an object. The sets of locations and objects form a Cartesian product; meaning that the elements of these sets form distinct ordered pairs, each pair made up of a location and an object. The specific geographic differentiation of information [related to general differentiation (in French: différenciation) but distinct from mathematical differentiation (in French: différentiation)] related to a spatial entity, concerns either the location, or the object, or both at once.

Geographic location-objects can be drawn on the walls of a cave, parchment, a sheet of paper, a computer screen, etc. This way of indicating their respective positions, their situation, makes it possible to construct a geomap, which is an artefact showing the relations between the location-objects represented. These drawings represent directly both differentiation by place and by object simultaneously. Historically, these drawings of geomaps came before maps, but they are still used in the form of various geographic diagrams: mental maps, advertisements, logos, computer graphics, cartograms, etc. While the situation on a geomap can be either qualitative or quantitative, the localization of a geographic location-object is achieved quantitatively using numerical coordinates in relation to axes in a plane. The graphic representation of each locus or object, using localization, is what is used to manufacture an artefact, called a map.

The object Earth can be seen as a set, considered to be a Whole. The constituent elements of that set, the Parts, sub-sets of the Whole, are geographic objects of the 1st order. When they are distinguished by a further property, the Parts of the Whole become geographic objects of the 2nd order. Clearly, further developments of this approach are going to generate Parts of successive orders (3, 4, .., n) depending on the distinctions made as a function of the problems under consideration. Then, each distinction leads to Parts of the Whole/s which may in turn be considered as Whole/s and subdivided into new Parts. If a distinction leads to differentiation, this latter leads to a spatial decomposition which generates classes of equivalence. The differentiation of the Whole into Parts can be interpreted as an equivalence (reflexive, symmetrical and transitive) or a tolerance relationship (reflexive and symmetrical, but not transitive). The geographic definition of the Whole/s and of the Parts does not imply any geodesic approach or any precise geometric figure (Nicolas and Marcus, 1997).

Every time a geovisualization is interpreted a posteriori using an a priori geographic vision, this is a combination of a geomap to produce a new geomap (figure 13). Therefore, in the case of the triangular-hexagonal geomap of the central places system, the place "centre" and the object "hexagon" are both differentiated. As a result, it is the "principles" attributed to the places in connection with their situation on the vertexes, the sides or inside the triangular-hexagonal objects, which explain the spatial relationships between location-objects. The hierarchical arrangement which emerges as a result is considered to be a natural or necessary order. Conversely, the map on which is overlaid the triangular-hexagonal geomap is only differentiated by object, in this case, the various geographic entities (functions, number of inhabitants, distances, etc.) which are involved in the populated areas. The localization of these depends on the projection systems which are defined a priori independently of the objects to be represented. It is not therefore the

cartographic location which explains the urban geographic properties of the location-objects under consideration, but their "geomap" graphic situation.

In practice, to verify if there is a match between the triangular-hexagonal image considered as a "model", and urban reality to validate the "centrality theory", an a priori geomap is overlaid onto an a posteriori map, considered to be an "outline map" or "base map". If it can be deduced that the "triangular-hexagonal model" is still applicable, even if it is reduced to a verbal metaphoric interpretation of the kind "everything seems to indicate that reality (ground truth) is in conformity with the model", then a new location-object with special characteristics is being fabricated (figure 13):

Reality = information → irregular polygons = form a posteriori,
Metaphor = centre → regular hexagon = form a priori.

The "centre" becomes a "symbolic place" of which all cartographic a posteriori representations—even if they are very or totally different from the a priori triangular-hexagonal representation—are acceptable proof of the theory, since, as Walter Christaller wrote: "Hence, the theory has a validity completely independent of what reality looks like, but only by virtue of its logic and the "sense of adequacy"" (Christaller, 1933). This assertion is reinforced by Peter Haggett for whom: "To ask for facts and nothing but facts" is to return to the "the anarchy [sic] of regional empiricism (Hagget, 1965) ". But it is a step too far when the omnipotence of "theory" justifies the fabrication of "exquisite corpses" to salvage a world where manipulation and institutional authority impose an understanding of the relationships between populated location-objects based entirely on a "natural" or "necessary" hierarchical central order.

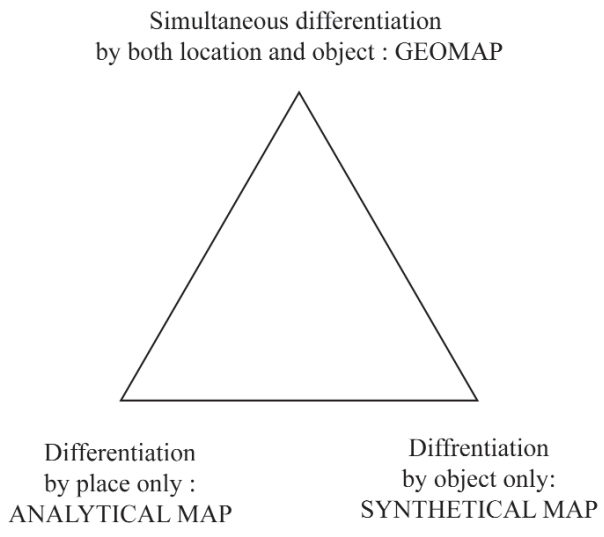



Figure 12: map and geomap

Despite sophisticated methods and a high degree of technical expertise, the results of form fabrication using geos-visualisations based on material supplied by geomatics and statistical data analysis are similarly subject to the constraints brought to bear by the relationships between a posteriori and a priori forms. Take the case of "cartograms", a new method for the presentation of statistical data recorded in political spatial entities (States) and their political or administrative subdivisions (regions, provinces, counties, etc.). They aim to put in the place of the traditional perception of the forms of States drawn according to the space they occupy on the continents, a new vision of these forms, the shape of which is distorted by the "weight" of the variables under consideration. This procedure, a contemporary mapmaking practice called "anamorphosis", has been in use since Antiquity, and is a play on perspective (Baltrusaitis, 1984). At the outset, it is supposed by convention that an observer looking at an image drawn on a plane surface in front of him, examines it from a viewpoint which allows him to visualise a circular portion of the artwork. It is therefore supposed that the eye of the beholder is situated at the summit of a cone, the circular base of which is what is being looked at. This point, called "vanishing point" in learned books on perspective, is perpendicular to the surface of the image. If the observer moves away from this perpendicular axis, his perspective is distorted depending on the direction of movement and the angle he is using. With reference to the so called "normal" perpendicular frontal vision, perpendicular vision from above is called "ceiling vision", and vision from below is "plunging". Finally, if two separate "vanishing points", spaced like two eyes, are used, vision is "bifocal" (Dalai Emiliani, 1968).

Maps, however, are manufactured with projection methods which give all those beholding them a "normal" vision, wherever they may be looking from. That being so, making a cartogram entails using a special type of anamorphosis. Instead of moving towards the top, the bottom or the sides, the user (whose point of view is supposed to be perpendicular to the map) is offered a modified form of the geographic entity seen in a way which depends on what is being shown. If the figures for a State's population is broken down into its administrative and political spatial entities - (the borough, the parish, etc.), those whose territory is "large" are shown with a "larger" surface if they have a large population, whereas those with few inhabitants end up with a "smaller" surface. The effect is identical if the spatial entity is "small": its gets "less small" or "smaller". As a result, in terms of area, the shape of large highly populated boroughs "grow" and squeeze out of shape those which are smaller or less populated. But, to avoid having the map "bursting out" in all directions, the external borders of the State are unchanged, so that its initial "shape" is retained, albeit deformed. Consequently, due to a "weighted cartographic transformation" (Cauvin, 1997; Cauvin, and Reymond, 1986), cartograms modify the surfaces of spatial entities so as to make them proportional to a quantitative variable but keeping them with a coherent Whole: the territory of the State concerned (Andrieu, 2005).

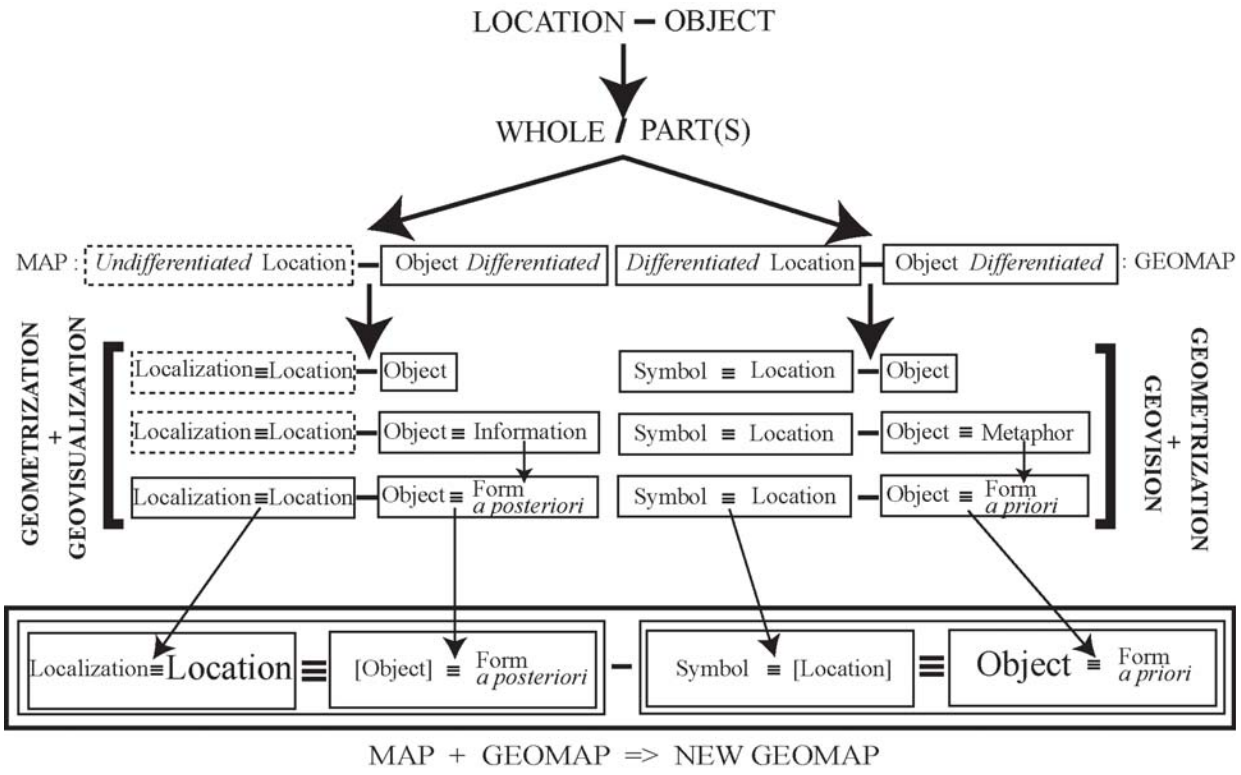



Figure 13: geointerpretation

To be more precise, a cartogram is manufactured using the "barycentre" (Bouvier, George and Le Lionnais, 1996) of the form of the spatial entity in which a numeric value for a measured variable has been entered, using an identical surface unit for the whole of the cartogram. Since, furthermore, the observer keeps a "normal" vision position for each spatial entity, perpendicular to the representation plane, the "centeredness" effect is reinforced. For those who favour cartograms, intuitive understanding of them is easier for an untutored observer, unused to working with ordinary maps, than it is for professional users. Even if this has not been verified by tests performed on a sufficient number of users, the cartogram promoters are continuing to use "hypercentration" to make them because they believe that this is scientifically justified. This "hypercentration" is also found elsewhere, not just in cartograms centred on a country such as France (Andrieu, 2005)[39], but also in cartograms "centred" on the world (Dodge, McDerby and Turner, 2008).

One of the more sophisticated methods for producing cartograms uses diffusion equations in molecular physics (Gastner and Newman, 2004). It was used to produce 366 cartograms using variables collected by several United Nations agencies (United Nation Development Program, World Health Organization, United Nations Statistics Divisions) in all the world's States (Newman and al., 2006). The starting point is a cylindrical equidistant projection map, the central axis of which is the Greenwich meridian (figure 14). The States represented individually are grouped into 12 subsets generating Whole/s by contiguity, although they do not constitute homogenous geopolitical units: Norway and Switzerland are included in Western Europe defined on the basis of the European Union, Turkey is part of Eastern Europe and Russia is in the Middle East with the Arab countries! Each variable is related to a State with a territory whose shape is deformed as a function of the absolute value of that variable. The result provides a visual comparison of the various States for each of the variables chosen.

A great deal of research would be possible using this considerable volume of material, all the more so since the Worldmapper website is free of access. Two of its creators used it to evaluate by comparison in what measure the equality in Article of the 1948 Universal Declaration of Human Rights: "All human beings are born free and equal in dignity and rights." is respected in today's world (Barford and Dorling, 2008). For the authors, this equality signifies that all over the surface of the Earth, men and women with equal ability, aptitude or competence should have equal chances, opportunities and respect. The variables are the following: 1) children (births, diseases, work, education) 2) gender equality (motherhood, contraception, employment); 3) work (agriculture, industry, services; 4) standard of living (daily purchasing power in US dollars); 5) travel (tourism, air passengers); 6) macro-economics (imports, exports, levies); 7) access to information (the Internet). All the cartograms reveal severe inequalities in contradiction with the equality set out in the Universal Declaration of Human Rights. Their conclusion is that: "Visualization [...] obliges us [English-speaking nations and

[39] Cartogram 6 for the presidential election in 2002 - the votes of the far right.

others where many have English as a second language] to consider what is corrupt, immoral and profane about how life has come to be so ordered, so cheap and so unjust." (Barford and Dorling, 2008). For the two writers, as for all the producers of Worldmapper, cartograms are therefore an objective and effective method of raising collective awareness, thanks to the "democratization of mapping"(Unwin, 2008). "Often our ideas about the world are based primarily on more nebulous material that might include stereotypes, news reports and personal accounts. These maps [cartograms] add to that and our imagination of the world because, rather than picking out a few stories of interest, they attempt to find a space for everyone living in the world. (Barford and Dorling, 2008) ".

That being so, as in the case of Christallerian centrality, there is in fact a conflict between an a priori geovision and an a posteriori geo-visualization. To verify this, we can try and imagine what shapes we would arrive at if "equal chances, opportunities and respect" were achieved: the initial shape of a State would coincide with the shape generated by the absolute value of the represented variable and there would be no distortions, or only minimal distortion, when changing variables. Therefore, there is indeed an a priori shape opposed in each cartogram to the a posteriori shape obtained by graphic processing. It is the "a posteriori abnormality" of the fact represented which deforms the a priori normality of the ideal. But the manner in which purchasing power is calculated gives a clear indication of what "normality" is. "In Indonesia US$ 10 buys more than it does in the United States, so comparing earning in US$ alone does not allow for the cost of living changing between places. The map shows purchasing power parity (PPP)—what someone earning PPP US$ 10 would buy in the United States "(Barford and Dorling, 2008). Cartograms 158 and 159 (figure 15) shows shapes which are all equally monstrous: on the one hand the abnormality of the "excessively rich" (United States: cartogram 158 and, on the other hand, the abnormality of the "excessively poor" (India: cartogram 179). While these considerations are in agreement with the authors' egalitarian ideals, it is not certain that they are in phase with the needs of the "excessively poor". They make their purchases where they are and not in the United States and they are more minded about the possibility of getting enough food than of buying goods at American prices. The generous way in which the authors set out the problems does indeed evidence well-documented scandalous injustice, but they are formulated in terms and in language which are primarily addressing English-speaking internet users, in other words, the "excessively rich".

The analogy between the dialectics of the shapes generated by the "Christallerian" representations on the one hand and the Worldmappers' representations is striking: 1) hypercentration of the representation; 2) opposition between the "ideal" geovision and the "real" geovizualisation. And yet, the "ideals" could hardly be more opposed: on the one side a pyramidal central hierarchic order with totalitarian excesses, on the other, an egalitarian central order with populist excesses.

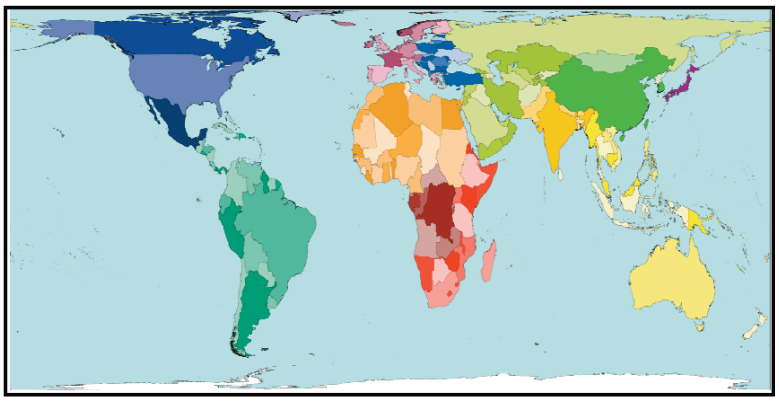
| CHRISTALLERIAN ORDER | WORLDMAPPER ORDER |
|---|---|
| Central | Central |
| Hierarchical | Egalitarian |
| Pyramidal | Flat |
| Evident | Evident |
| Normal/Abnormal | Normal/Abnormal |
| Ideal/Reality | Ideal/Reality |
| Totalitarian excesses | Populist excesses |

Table 3. Conclusion: Is the "Centre" a toxic concept in geography?

Not all the current "computer-graphics" methods experience such critical geo-interpretation problems, generated by the dialectic between shape geovisualizations and geovisions, as the Worldmapper cartograms. But none of them are entirely exempt from the dangers of determination or subversion of its shapes by geovisions, as in the case of the so-called "theory of centrality" or of the "central places system". In fact, as we have been recently reminded, adding coordinates to a table of data does not amount to adding two supplementary columns of variables: "Yet experience suggests that, although the techniques used might look much the same as those used in more general scientific visualization, there is actually something that is special about "geo" [...] but I suspect it is also to do with the ubiquitous presence in the real world of spatial autocorrelation [or] what, for want of a better word, I call "context (Unwin, 2008)." All the more since what is missing is a "well found theory to enable us to answer basic visualization questions such as "what works?" and even "what's likely to be the best way of displaying these data?". As a result, in the so-called "social" sciences, there is no theory with which to test the purely spatial theories using shapes drawn from "computer-graphics".

Because, contrary to what is generally stated, a "map" is not a "geomap". Today, in the majority of cases and contrary to what was done for many centuries, mapmaking generally precedes the production of geomaps and, furthermore, geomaps are overlaid onto "base maps". With the absence of any theory regarding the geographic significance of "computer visualization", there is the added confusion between cartography (which deals with differentiation by object) and geomapgraphy (which deals with differentiation by place and by object), so that the system is systematically skewed in favour of geovisions using places to the detriment of geovizualisation using localization.

Furthermore, the authoritative sway of very ancient metaphors and of their symbols in geovisions tends to paralyse critical faculties to such an extreme that there is blindness in the face of pseudo-scientific theories. A full half-century elapsed before the elementary mathematical errors made by Walter Christaller, August Lösch and Brian Joe Lobley Berry were discovered. How long will it be before are discovered those which may have slipped in to the sophisticated and mathematically complex procedure of "computer-graphics"? How many "exquisite corpses" will again be fabricated if the discourse of geographers continues to be poisoned by as toxic a concept as the "centre"?



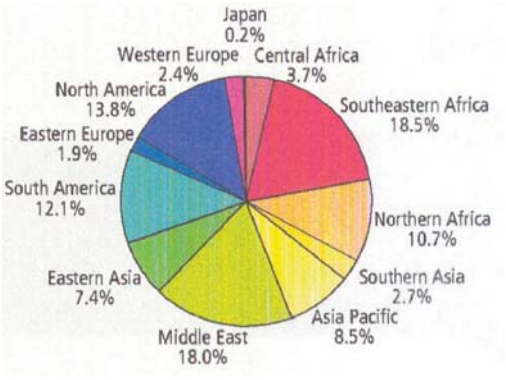
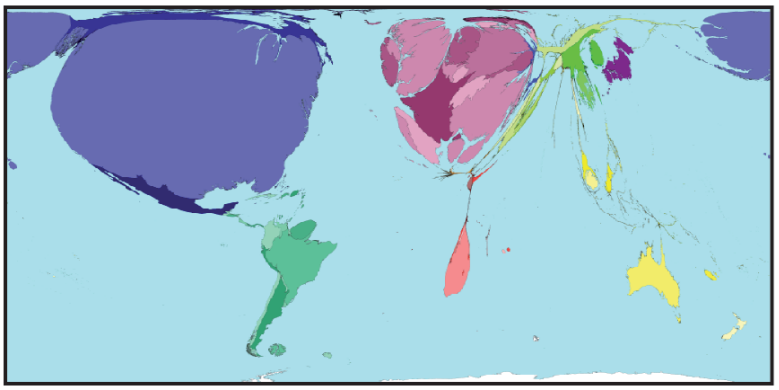


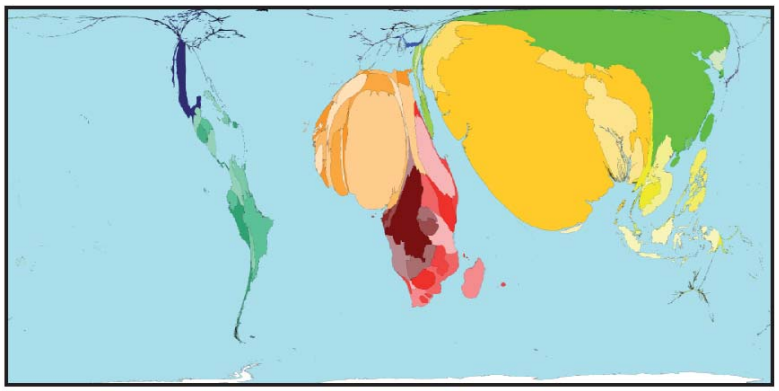

Figure 14: worldmapper: land area (map 1). Each territory's size on the map is drawn according to its land area.

*Worldmapper. The world as you've never seen it before.* Maps by Mark Newman, data by Danny Dorling, text by Anna Barford, quality control by Ben Wheeler, website by John Pritchard and poster design by Graham Allsopp.





Territory size shows the proportion of all people living on over PPP US$200 a day worldwide, that live there



Territory size shows the proportion of all people living on less than or equal to US$1 in purchasing power parity a day.

Figure 15: purchasing power (maps 158 and 179)

*Worldmapper. The world as you've never seen it before.* Maps by Mark Newman, data by Danny Dorling, text by Anna Barford, quality control by Ben Wheeler, website by John Pritchard and poster design by Graham Allsopp.

## REFERENCES


Andrieu, D. (2005). L'intérêt de l'usage des cartogrammes: l'exemple de la cartographie de l'élection présidentielle française de 2002. Mappemonde. 77(1), [On line] URL: http://www.webcitation.org/5lVSp5o0Y.

Aujac G. (1993). Claude Ptolémée astronome, astrologue, géographe: connaissance et représentation du monde habité. Paris, 427 pp.

Bailly, A. S. (1975). L'organisation urbaine. Théorie et modèles, Paris 272, pp.

Baltrusaitis, J. (1984). Anamorphoses, ou Thaumaturgus Opticus: les Perspectives dépravées. Paris.

Barford, A. & D. Dorling (2008). Telling an old story with new maps. in: Dodge, M., M. McDerby, & M. Turner. p. 67-107.

Baskin, C. W. (1966). Central places in southern Germany. New Jersey.

Bathelt, H. & J. Glückler (2003). Wirtschafts geographie. Ökonomische Beziehungen in räumlicher Perspektive. Stuttgart (2e édition remaniée).

Berry, B. J. L. (1956). Geographic aspects of the size and arrangement of urban centers: an examination of central place theory with an empirical test of the hypothesis of classes of central places. Washington.

Berry, B. J. L. (1967). Geography of market centers and retail distribution. New Jersey.

Blotevogel, H. H. (2002). Zum Verhältnis des Zentrale-Orte-Konzepts zu aktuellen gesellschaftspolitischen Grundsätzen und Zielsetzungen. in Blotevogel. H. H. p. 17-23.

Blotevogel, H. H. (2002). Fortentwicklung des Zentrale-Orte-Konzepts. Hannover.

Bobek, H. (1961-1978). Atlas der Republik Österreich. Vienne, 6 vol.

Bobek, H. & M. Fesl (1978). Das System der zentralen Orte Österreichs: eine empirische Untersuchung. Vienne.

Bobek, H. (1935). Eine neue Arbeit zur Stadtgeographie: Rezension von Walter Christaller. Die zentralen Orte in Süddeutschland. Zeitschrift der Gesellschaft für Erdkunde zu Berlin. p. 125-130.

Bobek, H. (1927). Grundfragen der Stadtgeographie. Geographischer Anzeiger. p. 213-224; republié dans Schöller, P. (1969). Allgemeine Stadtgeographie. Darmstadt, p. 195-219.

Bouvier, A., M. George & F. Le Lionnais (1996). Dictionnaire des mathématiques. Paris.

Böventer, E. (1963). Towards a united theory of spatial economic structure. Regional Science Association. Papers, X, Zürich Congress, p. 163-187.

Breton, A. & P. Éluard (1938). Dictionnaire abrégé du surréalisme. Paris, éd. 1969 (1ère édition: 1938) http://www.webcitation.org/5lVkyp6hv.

Brunet, R. (2000). Des modèles en géographie? Sens d'une recherche. Bulletin de la Société de Géographie de Liège, n°2, p. 21-30.

Burghardt, A. F. (1979). The origin of the road and city network of Roman Pannonia. Journal of Historical Geography, 5, 1, p. 1-20.

Capel, H. & J. L. Urteaga (1982). Las nuevas geografías. Barcelona.

Cauvin, C. (1997). Au sujet des transformations cartographiques de position. Cybergeo. Revue européenne de géographie, Cartographie, Imagerie, SIG, article 15, mis en ligne le 14 janvier 1997, modifié le 27 avril 2007. http://www.webcitation.org/5lVld6Vu6 Consultation: 22 décembre 2008.

Cauvin, C. & H. Reymond (1986). Nouvelles méthodes en cartographie. Montpellier.

CERTU (2001). Centralités dans la ville en mutation: quelles perspectives d'action pour les pouvoirs publics? Lyon.

Christaller, W. (1950). Das Grundgerüst der räumlichen Ordnung in Europa. Die Systeme der europäischen zentralen Orte. Frankfurter geographische Hefte, 24, 1, p. 10-19.

Christaller, W. (1933). Die zentralen Orte in Süddeutschland. Eine ökonomisch-geographische Untersuchung über die Gesetzmäßigkeit der Verbreitung und Entwicklung der Siedlungen mit städtischen Funktionen. Darmstadt, 1980 (1ère édition: Jena, 1933).

Comptes rendus du Congrès international de géographie d'Amsterdam, (1938). T. II, Travaux des sections. Section III a: Géographie humaine (Président: Prof. A. Demangeon). Séance du 21 juillet. Question 2: Rapports fonctionnels entre les agglomérations urbaines et les campagnes (Président: prof. A. Demangeon [Paris], président de séance: prof. C. Biermann [de Lausanne], remplaçant).

Clark, P. & B. Lepetit (1996). Capital cities and their hinterlands in early modern Europe. Aldershot; version révisée de Aerts, E. & P. Clark, Metropolitan cities and their Hinterlands in early modern Europe. Session B-6: Proceedings [of the] tenth International Economic History Congress, Leuven, August 1990, Louvain, 1990.

Dalai Emiliani, M. (1968). La question de la perspective. 1960-1968, Arte, 2 (republié in: Argan, G. C. & R. Wittkover (1990). Perspective et histoire au quattrocento. Montreuil, p. 97-117.



Davezies, L. (2008). La République et ses territoires. La circulation invisible des richesses, Paris.

Denzel, M. A. (1994). La practica della cambiatura. Europäischer Zahlungsverkehr vom 14. bis zum 17. Jahrhundert, Stuttgart.

Derks, H.(1986) Stad en Land. Markt en Oikos (I), Amsterdam.

Derks, H. (2001) Deutsche Westforschung. Ideologie und Praxis im 20. Jahrhundert, Leipzig.

Djament, G. & M. Covindassamy (2005). Traduire Christaller en français. Textes seuils, réception, récit de découverte, Cybergeo. Revue européenne de géographie, Epistémologie, Histoire, Didactique, article 298, mis en ligne le 25 janvier 2005, modifié le 11 mai 2007. http://www.webcitation.org/5lVms052L Consultation: 22 décembre 2008.

Dodge, M., M. McDerby, & M. Turner, (2008). Geographic visualization: Concepts, tools and applications. Chichester.

Dörries, H. (1934). Rezension von Walter Christaller. Die zentralen Orte in Süddeutschland.Geographische Zeitschrift, 40, p. 233-234.

Elmi, S. & C. Babin (1996). Histoire de la Terre. Paris (1ère éd.: 1994).

Favier, R. (1993). Les villes du Dauphiné aux XVIIe et XVIIIe siècles. Grenoble.

Fiétier, R. (1977). Histoire de la Franche-Comté. Toulouse.

Fray, J.-L. (2006). Villes et bourgs de Lorraine, réseaux urbains et centralité au Moyen Âge. Clermont-Ferrand.

Fittkau, D. (2004). Beeinflussung regionaler Kaufkraftströmen durch den Autobahnlückenschluß der A 49 Kassel-Gießen. Zur empirischen Relevanz der "New Economic Geography" in wirtschaftsgeographischen Fragestellungen. Göttingen.

Gastner Michael T. & Newman, M.E.J. (2004). Diffusion-based method for producing density equalizing maps. Proceedings of the National Academy of Sciences of the United States of America, 101 / 20, p. 7499-7504.

Gilomen, H. J. & M. Stercken, (2001). Zentren. Ausstrahlung, Einzugsbereich und Anziehungskraft von Städten und Siedlungen zwischen Rhein und Alpen. Zürich.

Gradmann, R. (1926). Volkstum und Rasse in Süddeutschland. Volk und Rasse, 1, p. 135-146.

Grimm, F.-D., G. Friedlein, & E. Müller (1997). Zentrensysteme in Mittel- und Osteuropa = Central place systems in Central and Eastern Europe. No 5.3 – MO1, Berlin, Stuttgart.

Güssefeldt, J. (2003). Empirische Aspekte einiger Modelle der. New Economic Geography. im Kontext jüngerer Entwicklungen des Einzelhandels. Die Erde, 134/1, p. 81-110.

Güssefeldt, J. (2005). Die Raumwirtschaftstheorien von Christaller und Lösch aus der Sicht von Wirtschaftsgeographie und. New Economic Geography. Göttingen.

Haggett, P. (1965). Locational analysis in human geography. Londres.

Haggett, P. (1983). Geography: a modern synthesis. New-York.

Haggett, P., A.D. Cliff & A. Frey (1977). Locational analysis: 1 Locational models, 2 Locational methods. Londres.

Heuwinkel, D. (2002). Niedersachsen. in: Blotevogel, H. H., p. 151-167 Huriot, J.-M., (1994) Von Thünen. Économie et espace, Paris.

Isard, W. *et al* (1960). Methods of regional analysis: an introduction to regional science. Cambridge Mass./ Londres.

Jordan, P. (1989). Atlas Ost- und Südosteuropa - Atlas of Eastern and Southeastern Europe. Berlin, Stuttgart.

Kegler, K. R., (2008). Walter Christaller. in Haar, I. & M. Fahlbusch, Handbuch der völkischen Wissenschaften. Personen—Institutionen—Forschungsprogramme—Stiftungen, München.

Krugman, P. (1995). Development, Geography, and Economic Theory. Cambridge, Massachusetts.

Kunow, J. (1988). Zentrale Orte in der Germania Inferior. Archäologisches Korrespondenzblatt, 18, p. 55-67.

Lang, B. (2002). Die Untergliederung der Bundesrepublik Deutschland in strukturierte Wirtschaftsregionen: eine empirische Untersuchung auf der Grundlage der Raumstrukturtheorien von von Thünen. Christaller und Lösch, Frankfurt a.M.

Lepetit, B. (1988). Les villes dans la France moderne (1740-1840). Paris.

Lösch, A. (1938). The nature of economic regions. The Southern Economic Journal, 5, p. 71-75.

Lösch, A. (1944). Die räumliche Ordnung der Wirtschaft. (1ère éd. Jena, 1940; 2e éd. remaniée, Jena, 1944); édition utilisée: Stuttgart, 1962 (reproduction du texte de 1944).

Mandelbrot, B. (1995). Les objets fractals: Forme, hasard et dimension. Paris.

Michalakis, M. & G. Nicolas (1986). Le cadavre exquis de la centralité. Eratosthène-Sphragide, 1, p. 38-87, Lausanne.

Miosga, M. (2002). Region München. in: Blotevogel, H. H., p. 143-150.

Newman, M. *et al.* (2006). Worldmapper. The world as you've never seen it before. © Copyright 2006 SASI Group (University of Sheffield) and Mark Newman (University of Michigan) http://www.webcitation.org/5lVuxjdl3. consultation: 2 décembre 2008.

Nicolas, G. & S. Marcus, (1997). Logique Tout/Partie. in Nicolas, G., Géographie(s) et langage(s). Interface, représentation, interdisciplinarité, Sion, p. 335-344. http://www.webcitation.org/5lVwRBd8O, rubrique: publications, e-Eratosthène, consultation: 23 novembre 2008.

Nicolas, G. & A. Radeff (2002). Décentralité/centralité: ordre ou désordre? in Gömmel, R. & M. A. Denzel, Weltwirtschaft und Wirtschaftordnung: Festschrift für Jürgen Schneider zum 65 Geburtstag. Stuttgart, p. 265-286.

Nicolas, G., A. Radeff, & S. Adam. En préparation: Hexagones et centres.

Ohji, T. (1986). Normative approach to the study of periodic markets in south India. Jimbu-Chiri (The human geography), 38, 4, p. 1-27.

Paelinck, J. H. P. (1988). L'équilibre général d'une économie stable. in: Ponsard, C., Analyse économique spatiale. Paris, p. 277-319.

Pinchemel, P. & G. Pinchernel (1988). La face de la Terre, Paris.

Platon, (1975). Œuvres complètes. Tome XI, Les lois, livre V (-370, -348), Paris, 3e éd.

Popper, K. W. (1959). The logic of scientific discovery. Londres.

Preston, R. E. (1992). Christaller's research on the geography of administrative areas. Progress in human geography. 16, 4, p. 523-539.

Pumain, D. & V. D. Leeuw (1998). La durabilité des systèmes spatiaux. in: Archeomedes, Des oppida aux métropoles. Archéologues et géographes en vallée du Rhône, Paris, p. 13-44.

Pumain, D. (2004). Christaller (modèle de). Hypergeo, consultation: 19 avril 2005 http://www.webcitation.org/5lVybZ1ju.

Pumain, D. (1982). La dynamique des villes. Paris.

Pumain, D., T. Paquot & R. Kleinschmager (2006). Dictionnaire. La ville et l'urbain, Paris.

Relieu, M. (1992) & M. Eliade (1907-1986). in: Encyclopédie philosophique universelle: Les œuvres philosophiques. Paris, t. 2, p. 3201-3204.

Reynaud, J.-E. (1841). Villes. in: Leroux, P.-H. & J.-E.Reynaud. Encyclopédie nouvelle ou dictionnaire philosophique, scientifique, littéraire et industriel offrant le tableau des connaissances humaines au XIXe siècle. Par une société de savants et de littérateurs, Paris, t. 8, p. 670-687.

Riegger, R. (1971). August Lösch. In memoriam, Heidenheim.

Robic, M.-C. (2001). Walter Christaller et la théorie des lieux centraux. Die zentralen Orte in Süddeutschland (1933). in: Lepetit, B. & C. Topalov. La ville dans les sciences sociales. Paris, p. 151-189 (texte) et p. 364-373 (notes).

Robson, B. T. (1973). Urban growth: an approach. Londres.

Rössler, M. & S. Schleiermacher (1993). Der Generalplan Ost. Hauptlinien der nationalsozialistischen Planungs- und Vernichtungspolitik. Berlin.

Rössler, M. (1988). Géographie et national-socialisme. L'espace géographique, 17/ 1, p. 5-14.

Rössler, M. (1990). Wissenschaft und Lebensraum. Geographische Ostforschung im Nationalsozialismus. Berlin/Hamburg.

Sauberer, M., V. Surd & E. Tomasi (1990). Ausstattung der ländlichen Siedlungen in Siebenbürgen mit zentralen Einrichtungen = Availability of central facilities in rural settlements of Transylvania. No 5.2 – R2, Berlin, Stuttgart.

Short, J. R. (1996). The urban order: An introduction to cities, culture, and power. Malden USA/ Oxford UK.

Sombart, W. (1930). Die drei Nationalökonomien. Geschichte und System der Lehre von der Wirtschaft. München und Leipzig.

Staack, J. (1995). Die Klassifikation deutscher Städte nach ihrer regionalen Zentralität. Frankfurt a.M. etc.

Stolper, W. F. (1954). August Lösch in Memoriam. in: Woglom, W. H., p. VII-XI.

Thünen, J. H., (1826-1875). Der isolierte Staat in Beziehung auf Landwirtschaft und Nationalökonomie. Hamburg, Rostock et Berlin.

Ullman, E. (1941). A theory of location for cities. American journal of sociology, 56, p. 835-864.

Unwin, D. J. (2008). Foreword: encounters with "(geo) visualization". in: Dodge, M., M. McDerby & M. Turner, éds (2008). p. XI-XVI.

Vagaggini, V. & G. Dematteis (1976). I metodi analitici della geografia. Florence.

Vanagas, J. (2003). Miesto Teorija [Théorie urbaine]. Vilnius.

Weichhart, P., H. Fassmann & W. Hesina (2005). Zentralität und Raumentwicklung. Vienne.

Wikipedia. http://www.webcitation.org/5lXbzdcbj, consultation: 27 décembre 2008.

Woessner, R. (2008). La métropole Rhin Rhône: vers l'émergence d'un territoire. Colmar.

Woglom, W. H. (1954). The economics of location, New Haven. Traduction du texte de Lösch, A. (1944).

Zipf, G. K. (1949). Human behaviour and the principle of least effort. An introduction to human ecology. Cambridge (Massachusetts).